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REACTOR TECHNOLOGY

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Contents

REVIEW ARTICLES

FAST BREEDER PROGRAM IN EUROPE
Aldo Forcella, Nuclear Associates International Corporation 1

GAS-TURBINE NUCLEAR POWER PLANTS
A. A. Katterhenry, Southern Nuclear Engineering, Inc. 7

THE OUTLOOK FOR NUCLEAR POWER-STATION CAPITAL COSTS
William Steigelmann, Franklin Institute Research Laboratories 14

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ON THE DEVELOPMENT OF FAST BREEDERS
W. Häfele, Institut für Angewandte Reaktorphysik, Kernforschungszentrum Karlsruhe 18

SYSTEM AND COMPONENT DESIGN FOR FAST REACTORS
L. J. Koch, Argonne National Laboratory 36

FUEL CONSIDERATIONS
F. Sebilleau, Commissariat à l'Énergie Atomique 54

FAST REACTOR CORE HEAT REMOVAL
Dieter Smidt, Institut für Reaktorentwicklung, Kernforschungszentrum Karlsruhe 58

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CURRENT AWARENESS REVIEW

VARIATIONAL TECHNIQUES AS A METHOD FOR MULTIDIMENSIONAL REACTOR CALCULATIONS
Myrna L. Steele, USAEC Division of Technical Information Extension 73

*Formerly *Reactor and Fuel-Processing Technology.*

Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402, $3.00 per year.

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Fast Breeder Reactor Program in Europe

By Aldo Forcella*

The principal motivation for the existence of several high-priority fast breeder reactor development programs in Western Europe is the strategy of using bred plutonium to reduce the future dependency of the European power industry on the importation of uranium ore.

This strategy can be justified only by the early insertion of breeder reactors into the electrical network of Western Europe. Plutonium accumulation from present and near-future thermal reactors would be used to fuel fast breeders in the 1980's, thereby making substantial inroads to ore requirements in the 1990's and beyond.

The priority assigned to the fast breeder programs derives from the need under this strategy for early large-scale exploitation of plutonium breeding. If the technical targets and program time schedules implied by this strategy could not be met, an alternate strategy having less attractive overall nuclear power economics should perhaps have been chosen.

The above considerations generated enough incentives to national and international agencies and industries to assign high priority to the programs and to invest large resources in fast breeder reactor development.

There are now four large fast breeder programs in operation in Western Europe. They all have in common two main features: sodium primary and secondary cooling systems and mixed-oxide fuel with an option on carbides at a later date. Table 1 gives the geographic distribution of the main facilities of these programs.

Besides the countries listed in Table 1, which are heavily committed to development of sodium-cooled fast breeders, there is a growing interest in other nations for an alternate line of development around the gas-cooled breeder reactor concept.[1]

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PROTOTYPES

Three sodium-cooled fast breeder prototype plants listed in Table 2 are in various phases of construction.

The PFR[2] and Phenix[3] reactors are of the pool type or integrated design where all the primary equipment, including the intermediate heat exchangers (IHX's), is contained in a sodium primary vessel. Primary sodium is pumped to the lower plenum, flows up the fuel elements, and flows down in the sodium-to-sodium intermediate heat exchangers housed in the reactor jacket (separating the hot from the cold sodium in the primary vessel). Both reactors have three primary pumps (vertical, centrifugal, and sodium bearing), and six IHX's. Simplicity, compactness, and high cooling safety are claimed as the main reasons for the adoption of the "pool" design. The SNR reactor[4] has a more traditional three primary-loops system.

The main building and civil work for the PFR have been completed at Dounreay.[5] Site fabrication of the primary vessel and its associated leak jacket has been completed, and shop work on all the other main components is progressing. The assembly of these components is due to be completed at the end of 1971. Acceptance tests, including a dynamic water test of the primary circuit, are scheduled to begin in 1972. A 1-year delay in the construction schedule of the PFR was caused by defects detected during fabrication of the reactor "roof," a steel structure composed of an outer ring girder connected by spokes to the central section in which the rotating shield plug operates.

Civil work for the Phenix reactor at Marcoule, France, is under way,[6] and many components are in fabrication (turbogenerator set, reactor vessels, grid, primary and secondary pumps, etc.). Orders will soon be placed for the rotating shield plug, IHX's, sodium boilers, and secondary and auxiliary systems.

SODIUM-COMPONENTS DEVELOPMENT

Presently sodium-components development is mostly in support of the prototype designs. In the United Kingdom,[7] the English Electric Company's

Table 1 Main Facilities Within the European Fast Breeder Development Programs

	Benelux–Federal Republic of Germany	France	Italy	United Kingdom
Fast test reactor	KNK II, 20 MW(e)	Rapsodie, 24 MW(t)	PEC, 140 MW(t)	DFR, 14 MW(e)
Operation start	1972	1967	1974	1959
Location	Karlsruhe, FRG	Cadarache	Brasimore	Dounreay
Prototype reactor	SNR, 300 MW(e)	Phenix, 250 MW(e)		PFR, 250 MW(e)
Operation start	1974 to 1975	1973		1972
Location	Weisweiler, FRG	Marcoule		Dounreay
Large critical	Sneak	Masurca		Zebra
Operation start	1966	1964		1962
Location	Karlsruhe, FRG	Cadarache		Winfrith
Large sodium IHX and boiler testing	70/50 MW(t)	50 MW(t)	50 MW(t)	
Operation start	1970	1970	1971	
Location	Hengelo, The Netherlands	Renardieres	Casaccia	
Large sodium pump testing	APB			
Operation start	1969			
Location	Bensberg			
Plutonium fuel-element fabrication	ALKEM*	CEA		UKAEA*
Operation start	1970	1966		1970
Location	Wolfgang b/Hanau	Cadarache		Windscale

*Also thermal recycle fuel.

Table 2 Sodium-Cooled Fast Breeder Prototype Plants

Name	Location	Owners	Startup	Power, MW(e)	Sodium Outlet, °C	Core fuel
PFR	Dounreay, United Kingdom	UKAEA	1972	250	560 to 600	PuO_2–UO_2
Phenix	Marcoule, France	CEA–EDF	1973	250	560 to 590	PuO_2–UO_2
SNR	Weisweiler, Federal Republic of Germany	FRG–Benelux	1974 to 1975	295	550	PuO_2–UO_2

Reactor Equipment Division has in operation a number of sodium loops where many components, such as large butterfly valves of 580 mm bore, a-c and d-c electromagnetic pumps and flowmeters, plugging meters, level probes, small valves, and special selector valves, are performance tested. A general-purpose sodium facility provides a large vessel in which a variety of experiments can be carried out in either clean sodium at temperatures up to 650°C or in an argon–sodium vapor environment of the reactor blanket gas. The main loop consists of a dump tank of 680 liters capacity and a test vessel. This loop is used for the development of small components, as well as for more fundamental work on friction, wear, and static adhesion in sodium and sodium vapor.

A second but larger loop similar to the one mentioned above is used for the development and calibration of larger items, such as pumps and flowmeters, under pressures up to 6 kg/cm^2, temperatures up to 650°C, and flows up to 68 m^3/hr. Designs for PFR primary and secondary pumps have been checked using a half-scale model impeller in water.

At Fairey Engineering Company's facilities, a general-purpose sodium loop is used to perform work on sodium environments. This versatile, high-temperature loop has four test points into which various sodium experiments can be fitted. The main circuit can operate at 400°C and 2.8 kg/cm^2. This loop provides 680 kg of sodium having an operating oxide level of 5 ppm oxygen to the test sections, which have their own heaters for raising temperatures well above 400°C.

New sodium facilities have been installed at Babcock & Wilcox Ltd.'s Research Station at Renfrew in a specially built sodium hall.

Many alternatives in PFR sodium-boiler design were studied in the light of test results from the sodium–water reaction test facility Noah. The following design characteristics were adopted:

1. Elimination of tube welds in the sodium. In general, it was felt that these welds are more frequently associated with faults leading to leaks than the parent tube material is.

2. A forced-circulation system with a circulation rate chosen to prevent local dry-out. This arrangement avoids the effects of frequent wetting and drying and thermal cycling associated with "once-through boilers."

3. U-shaped tubes having one bend per tube so as to minimize defects in the tubes.

4. Tube–tube plate welds carried out by a bore-welding technique that gives crevice-free welds which can be inspected.

5. Use of a sodium flow path that is substantially parallel to the tubes so that the possibility of serious tube vibration due to vortex shedding associated with crossflow is virtually eliminated. Fretting and fatigue leading to tube failure were considered to be least likely with sodium flow along the tubes.

6. Provision for unbolting and removing from the shell a complete boiler section consisting of tubes, tube plates, and headers so that minor modifications or repairs can be carried out or a replacement bundle fitted.

"Once-through" sodium boilers will be the object of a major development effort for application to future commercial fast breeders in the United Kingdom.

In France, sodium-components development[6] for the Phenix 250-MW(e) prototype is supported by a program involving endurance and reliability testing of partial or full-scale component prototypes in a sodium environment. This testing program is similar to that used previously for the Rapsodie reactor.

Particular efforts were devoted to establishing a design of the sodium boilers. A 5-MW(t) sodium loop at Grand Quevilly near Rouen has been used extensively since 1964. First a reduced scale model of a double-wall tube boiler was tested; then a module section of a single-wall tube of the present Phenix plant design was subjected to an endurance test from December 1967 until March 1969. Safety problems deriving from sodium–water reaction on this boiler design are being investigated on a special loop at Cadarache. The present Phenix boiler design developed by the Stein Industries Company consists of a 12-module system comprising seven steam-tube clusters inside a sodium sheath bent in single-plane coils. By 1970 a boiler test station will be completed at EDF's Centre des Renardieres. This facility is equipped with a 50-MW(t) sodium heater. Three full-scale models of the Phenix boilers will undergo final test runs there. Other boiler designs for the later commercial-size breeder reactors will be tested at the Renardieres Station.

Guinard Company and Hispano Suiza Company have developed sodium mechanical pumps having up to 4000 m^3/hr capacity. The GAAA Company has wide manufacturing experience in auxiliary sodium equipment such as cold traps, plugging meters, flowmeters, hydrogen detectors, and electromagnetic pumps.

For support[4,8,9] of the Federal Republic of Germany–Benelux SNR 300-MW(e) prototype breeder project, sodium-components development started at the Interatom Company's Bensberg site on a 5-MW(t) heat-transfer test facility. This operation began in 1965 in connection with the KNK thermal sodium reactor project. Tested were IHX's and sodium-boiler designs that allowed steam conditions of 510°C and 80 atm. Other sections of this facility are for stop- and control-valve tests; thermal-shock tests; and reactor-component, sodium-tank, and steam-tube rupture tests. Since the inception of the fast breeder program, modifications have been made to raise the steam conditions to 540°C and 200 atm.

A large sodium-pump test facility, APB, is nearing completion (end of 1969) at Bensberg. Here, sodium pumps up to 15,000 m^3/hr capacity can be tested. Tests on full-scale SNR-type pumps of about 5000 m^3/hr capacity will be run first. This loop will have 5-MW coolers. Provisions will be made for thermal-shock tests.

Another large facility, ASB, for investigation of safety devices for large sodium boilers (release of sodium–water reaction products in the atmosphere) has been in operation at Bensberg since 1968. Here, experiments on the effects of pinholes, pressure, and

temperature transients during sodium–water reactions will be conducted. Reliability of pressure-release systems and separation devices will also be checked at this facility.

At Hengelo in The Netherlands, the Neratoom Company is constructing a 50-MW(t) sodium IHX and boiler test station. Maximum steam temperatures allowed will be 600°C. Operation is scheduled for 1970.

In Italy[10] a 1-MW loop for testing of sodium-boiler tube-bundle sections is nearing completion at CNEN's Casaccia laboratory. Two loops are in operation for investigation of sodium-purification systems at Casaccia. Experiments on sodium–water reactions are carried out at CNEN's Brasimone site. A 50-MW(t) IHX and sodium-boiler test facility is also planned to be built at Casaccia in collaboration with AMN Company and PMN Company. This facility is scheduled to go into operation in 1971.

FUEL DEVELOPMENT

As mentioned above, all four European programs concentrate on mixed plutonium and uranium oxide fuel-pin-type elements for both prototype and early-target commercial breeders.

In France[6] the high availability of the Rapsodie reactor has contributed to the obtaining of much information from high-level-irradiation tests. Two irradiation experiments, Orphee and Aurore, aimed at testing of Phenix-type fuel pins under close to operating conditions are under way: as of July 1969 two Aurore assemblies had reached 66,000 MWd/metric ton. By spring 1970 a new core will be loaded in Rapsodie and will raise the central neutron flux from 1.9×10^{15} to 3×10^{15} neutrons/(cm^2)(sec). As a result the French CEA can obtain valuable information sooner on pin-cladding voidage at doses of 1×10^{23} neutrons/cm^2. Irradiation of oxide and carbide pins with linear heat ratings of 600 W/cm (for oxides) and 1000 W/cm (for carbides) will also be performed. Structural-materials irradiation will be performed in the core central region. Table 3 gives the irradiation levels reached by mixed-oxide pins in the Rapsodie reactor as of July 1969. Postirradiation examination[11] indicated that fission-gas release of up to 55% and changes in cladding properties appear to be the major problems. Xenon activity in Rapsodie reactor blanket gas also revealed the existence of a previously undetected pin-cladding failure.

The French CEA has facilities for plutonium-fuel-element fabrication and nondestructive postirradiation at Cadarache and another plant for postirradiation examination at Fontenay-aux-Roses.

Table 3 Burnup of Mixed-Oxide Pins in the Rapsodie Reactor[6]

Number of pins	Burnup, MWd/metric ton
282	Up to 10,000
511	10,000 to 20,000
553	20,000 to 30,000
1383	30,000 to 40,000
955	40,000 to 50,000
256	50,000 to 60,000
54	60,000 to 66,000

Up to October 1969 (Ref. 12), several fuel subassemblies consisting of 77 mixed-oxide pins had been irradiated in the Dounreay DFR in Britain to maximum burnups varying from 5.1 to 7.3%; maximum cladding mid-wall temperatures of about 700°C were experienced. The pins are 0.2 in. in diameter as in the PFR prototype. Similar single-pin experiments have been successfully taken to burnups of up to 10%. Further development is in progress to investigate the potential of these pins (now clad with 316 stainless steel) by using thicker and alternate cladding materials and by modifying the present specifications to reduce fabrication costs. A major concern is the effect of material voidage at high fast-neutron doses on fuel-pin endurance and structural material in the primary vessel through the reactor lifetime. Until material doses of 1×10^{23} neutrons/cm^2 are achieved, it will not be possible to know whether there is a saturation damage point or whether the effect is cumulative with the dose.

The PFR plant is scheduled to be at power during 1972, and its target irradiation of 7.5% maximum burnup should be achieved about 1 year later. Variants in the present PFR fuel-element design, largely aimed at removing fuel-cladding distention characteristics associated with fuel swelling and cladding voidage, will be tested in the PFR first core, along with more advanced fuel designs that could lead to lower fuel-cycle costs in the commercial fast breeders of the future.

Several experimental[13] and pilot-plant facilities for plutonium fuels exist within the UKAEA. A major plant for fast reactor plutonium-fuel-element production is nearing completion at Windscale. This plant design is based on the following design criteria: (1) continuous operation, (2) geometrically safe units, and (3) remote operation using a sealed-face system.

In the Federal Republic of Germany,[8,14] plutonium fuel fabrication is being investigated at the Karlsruhe Center. This laboratory also provides fuel assemblies for irradiation tests. The European Institute of Transuranium Elements, which is located at Karlsruhe, is also engaged in fuel development for the fast breeder program. The companies ALKEM and RBG are responsible for the development of fabrication processes and production.

There is no fast irradiation facility in the Federal Republic of Germany. The 20-MW(e) KNK reactor at Karlsruhe, a compact thermal sodium-cooled reactor due to begin operation in 1970, will have a central fast region (SNR-type fuel) and an outer zirconium hydride region when core 2 is loaded in 1972 (KNK II). Meanwhile GFK Karlsruhe is collaborating with the Mol Center in Belgium and the Petten Center of The Netherlands in an irradiation test program utilizing many reactors of the world. In this program, liquid-metal loops were screened from thermal neutrons to provide a simulated fast reactor neutron-flux region in the BR-2 MTR at Mol. Also, these collaborators now share with CEA of France a 77-pin test assembly in the DFR at Dounreay, and more tests are planned in both the Enrico Fermi Fast Breeder Reactor in the United States and the Rapsodie reactor in France. A decision may be taken to build a large fast breeder reactor (FR-3) in the Federal Republic of Germany, and high fluxes and linear heat ratings could be available in that reactor for testing oxide and carbide fuels.

Italy has a limited irradiation program under way, but the PEC Fast Test Reactor,[15] which is scheduled to be in operation in 1974, will allow fuel testing under advanced operating conditions: maximum flux of 3.5×10^{15} neutrons/(cm^2)(sec) and extremely high linear heat rates in three independent test loops. An out-of-pile loop gave encouraging results in the testing of a fuel-venting device.[29]

In other work on fast breeder development:

1. Several loops for studies of sodium corrosion and mass transfer are in operation.
2. Austenitic stainless steels and vanadium-base alloys are being widely investigated as cladding for oxide and carbide fuels.
3. Effects of cold work[30] on various compositions of stainless steels that have been subjected to high-temperature and irradiation damage are currently being investigated.
4. Creep-strength and stress-rupture experiments are being conducted.

Analyses based on experimental investigations are being carried out extensively in out-of-pile tests within the various programs. Examples are (1) overall mechanical behavior of fuel-element and control-rod mock-ups in long-term endurance tests and in thermal-cycling and thermal-shock studies with particular reference to pin-spacer design, (2) thermal and hydraulic studies oriented mostly toward thermal mixing, hot-channel factors, and gas entrainment, and (3) safety studies of accidents involving sodium boiling and/or fuel meltdown, particularly in connection with local plugging.

REACTOR PHYSICS

In fast critical facilities such as Zebra[16] in Britain, Masurca in France, Sneak[17] in the Federal Republic of Germany, and FR-0 in Sweden, various assembly configurations are being used to obtain, for the various fast breeder programs, physics design data, breeding-gain values of various reactor configurations, and improved calculation methods. These facilities are also being used in studying larger cores and in reducing the uncertainties associated with the Doppler coefficient, the sodium-void coefficient, and the neutron balances. Other zero-power critical and subcritical facilities being utilized are Stark and Suak in the Federal Republic of Germany, Vera in the United Kingdom, and Minerve and the Harmonie Source Reactor in France.

Zebra assemblies 5 and 7 (PFR mock-ups) produced measurement results[18] in good agreement with the calculated values of control-rod worths and sodium-void reactivities. More adjustments[17-19] of data sets are required as a consequence of recent measurements in various critical assemblies of Doppler coefficients for ^{238}U and reactivity worths of fissile isotopes.

Critical and subcritical experiments specifically designed for testing data sets[20-23] have been performed at various European sites. Methods of adjusting data to match integral results[24-26] are leading to more accurate revisions of data libraries in this domain.

GFK Karlsruhe and Euratom are participating in the SEFOR Program in the United States. Fuel-management codes for two- and three-dimensional triangular geometries have already been developed[27,28] for PFR, Phenix, and SNR reactor operation.

FUEL REPROCESSING

Several problems arising in handling and reprocessing of very-high-burnup fast reactor fuels are being investigated and checked.

Thus the Rapsodie reactor fuel-reprocessing scheme was first drawn up at the Hot Chemical Cell Facility at Fontenay-aux-Roses.[31] Later a special reprocessing facility for fast reactor fuels was built at La Hague.

A laboratory-scale plant was built at Karlsruhe to test aqueous reprocessing of fast reactor fuels having burnups of up to 100,000 MWd/ton. This plant has a capacity of about 1 kg/day.

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Gas-Turbine Nuclear Power Plants

By A. A. Katterhenry*

Power plants using gas-cooled reactors have generated almost two-thirds of all the electric power produced by nuclear generating stations in the free world. This illustrates the broad background of experience with gas-cooled reactors and the extensive development they have undergone. There are three distinct generations of gas-cooled power reactors, all graphite moderated: (1) the Magnox reactors, which are fueled with natural-uranium metal clad in a magnesium alloy; (2) the advanced gas-cooled reactors (AGR's), which use enriched-UO_2 fuel clad in stainless steel; and (3) the high-temperature gas-cooled reactors (HTGR's), whose fuel is UO_2 or UC particles with various coatings.

The Magnox reactors are British or French designs and are used in the national power grids of those countries; the British have also exported two Magnox units—one to Italy and the other to Japan. Attention has now turned to later generations of gas-cooled reactors in Great Britain, and France has dropped gas cooling—at least in its present plans for expanding nuclear generating capacity.

In Britain an AGR has been in operation since 1963; four additional plants, with a total capacity of 5000 MW(e), are currently under construction, and another 2500 MW(e) of AGR capacity is planned. The third-generation concept, the HTGR, is seen in four plants that are now in operation: Dragon in Great Britain, Peach Bottom and UHTREX in the United States, and the AVR in West Germany. The power ratings of these reactors are 20, 115, 3, and 46 MW(t), respectively. Construction is under way on the 330-MW(e) Fort St. Vrain plant in the United States and the 25-MW(e) KSH plant in Germany. A 300-MW(e) plant known as the THTR is planned for Germany.

The AGR coolant is carbon dioxide; its temperature at the reactor outlet is about 1200°F. The HTGR's use helium as the coolant, with core outlet temperatures of 1300 to 1400°F (except in UHTREX). UHTREX—the Ultrahigh-Temperature Reactor Experiment—was designed to operate with a coolant temperature at the reactor outlet of 2400°F, a goal which has recently been reached. With either reactor type, more-or-less conventional steam cycles have been used, with the steam temperature about 1000°F at the turbine inlet. Most large conventional (fossil-fueled) units now under construction in the United States are designed to operate with steam temperatures at the turbine throttle of 1000°F and with pressures of either 2400 to 2600 psi or 3500 to 3700 psi at the same point. Experience has shown that steam temperatures above 1000 to 1050°F are not economically feasible owing to materials limitations. The same considerations apply in selecting the steam temperature in gas-cooled reactor plants.

THE GAS-TURBINE POWER CYCLE

As an alternate to a steam power cycle, the gas-turbine power cycle is being studied seriously for use with gas-cooled reactors. Advantages of a plant envisioned for this concept, as compared with a steam-cycle plant, are as follows:

1. The high-temperature capability of the reactor could be better utilized in a gas-turbine circuit. Maximum gas pressures would not need to be nearly as high as the steam pressure in an efficient steam cycle, and thermal efficiency would be as high as that of a conventional steam cycle, or even higher.

2. The plant would have a direct cycle. The reactor coolant would also be the fluid in the power-generating portion of the cycle, eliminating the presently used steam circuit and steam generators. The entire plant would be simpler, and this should result in savings in capital cost.

3. Heat would be rejected at higher temperatures than from steam plants. Waste heat could be used as process heat, or the higher temperatures could make air cooling feasible, thereby avoiding thermal effects on bodies of water.

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Against these advantages, the gas-turbine power cycle would have some disadvantages:

1. The reactor coolant is the turbine fluid, so there could be radiation problems during operation and maintenance.
2. Turbo-machinery, including bearings and seals, has not been developed, although development has begun.
3. Load pickup would be slower than with most other plant types, and low-load operation would be inefficient.

Figure 1 shows a simplified flow diagram of a gas-turbine nuclear power plant. A Brayton cycle is

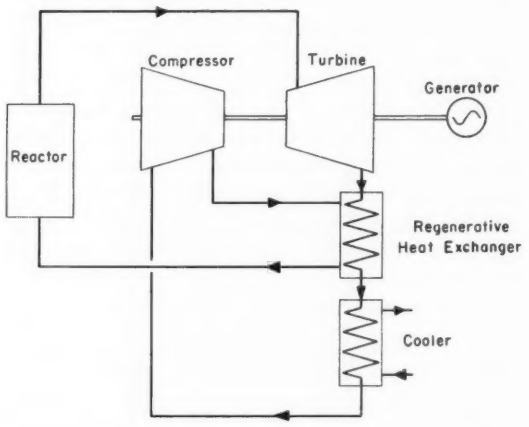

Fig. 1 Gas-turbine nuclear power cycle.

used. Hot high-pressure gas from the reactor enters the turbine, where it expands to lower pressure and causes the turbine to work. After leaving the turbine, the gas is cooled in a regenerative heat exchanger and then in a precooler. The cold low-pressure gas is compressed by the compressor, which is driven by the turbine. The cycle is completed when the high-pressure gas from the compressor is heated in the regenerative heat exchanger and then in the reactor. The regenerative heat exchanger recovers heat from the turbine exhaust which would otherwise be wasted, and thus the cycle efficiency is increased. The cycle is thermodynamically similar to those used in conventional (fossil-fueled) gas turbines except that the regenerative heat exchanger is frequently omitted in the conventional units, an omission that results in poorer efficiency. Conventional gas turbines commonly operate with gas temperatures in the range 1500 to 1800°F or higher, and at these temperatures very high thermal efficiencies are possible if waste heat is recovered.

THE ML-1 GAS-TURBINE PLANT

At least one gas-turbine-cycle plant has been built and test operated—the U. S. Army's ML-1. The ML-1 program is summarized in Ref. 1. The ML-1 was a 400-kW(e) unit intended to be used as a field power generator. The unit was required to be lightweight, compact, and air-transportable, and its design was optimized for these qualities rather than thermal efficiency or economics. The cycle (cooling) fluid of the water-moderated reactor was nitrogen during the test period, with air to be ultimately used in the field. The design turbine inlet temperature was 1200°F. The unit was operated during 1962 to 1965. The project was terminated in 1966 prior to achievement of full design performance.

THE KSH GAS-TURBINE PLANT

To demonstrate the feasibility of a direct gas-turbine cycle for a commercial central-station power plant, the Federal Republic of Germany is building the 25-MW(e) KSH unit near Geesthacht in Schleswig-Holstein. The KSH will use an HTGR with helium as the coolant and is scheduled for 1973 operation. References 2 and 3 describe the plant.

The KSH is unique in that it will use an existing reactor concept and technology in a new power-cycle configuration, whereas most previous work has been on development of new nuclear technology for use with an existing power cycle. Principal areas of investigation in the project will be the effects of fission products in the coolant; dynamic behavior and control; engineered safeguards and safety analysis; and the design, construction, and operation of the turbine.

Power Cycle

Figure 2 is the flow diagram of the main power cycle of KSH and is typical of those for the larger plants which have been proposed. Valves V-1 and V-2 on Fig. 2 are control valves and are closed except during part-load operation. Figure 3 is the h–s (enthalpy–entropy) diagram of the KSH cycle, which is a modified Brayton cycle, with both regeneration and intercooling. The hot gas (helium) leaving the reactor expands in the turbine (A to B on Fig. 3) and is then cooled, first in the regenerative heat exchanger (B to C) and next in the precooler (C to D). The helium then passes through the three-stage compressor (D to

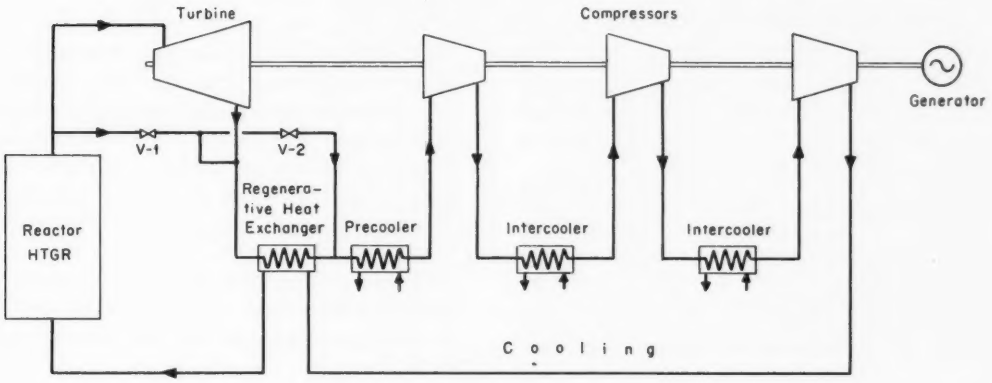

Fig. 2 The KSH power cycle.

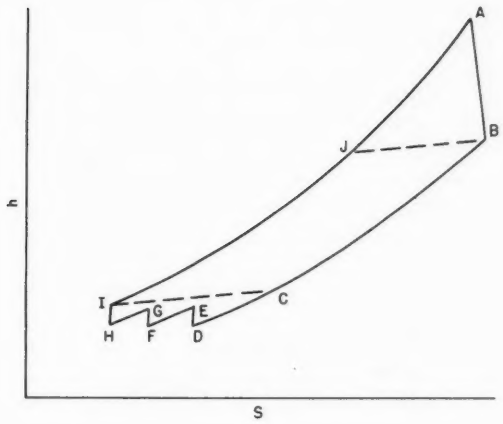

Fig. 3 The $h-s$ diagram of the KSH cycle.

E, F to G, and H to I), with intercooling between stages (E to F and G to H). Finally the compressed gas is heated in the regenerative heat exchanger (I to J) and then in the reactor (J to A). The broken lines on the $h-s$ diagram indicate the temperatures at the regenerative heat-exchanger terminals (B, C, I, and J). In an ideal cycle the expansion and compression processes would be adiabatic and the heating and cooling processes would be at constant pressure. Only about half the energy extracted in the turbine during the expansion is available to drive the generator, with the balance required to drive the compressor.

Reactor

The reactor in the KSH plant is quite similar in design to the Peach Bottom HTGR,[4] which is now operating. The basic core element in both reactors is a graphite cylinder 3½ in. in diameter. The cylinder is divided axially into three sections, with fuel contained only in the center section.

The cylindrical elements are arranged in triangular array (as viewed from above) with their axes oriented vertically. In KSH, helium flows in the tricuspid openings formed by the fuel elements, as in the Peach Bottom reactor. Graphite control-rod guide tubes, also 3½ in. in diameter, replace some of the fuel elements in both reactors.

The unfueled end segments of the fuel elements form axial reflectors, and unfueled graphite cylinders around the outside of the core make a radial reflector. In both reactors the fuel is highly enriched uranium, with thorium fertile material, and is located in an annular zone in each core element. The fuel and fertile materials are in the form of coated particles, the particles being oxides in KSH and carbides in Peach Bottom. Each Peach Bottom fuel element is designed to have a small internal purge gas flow and a fission-product trap. However, these provisions are not included at KSH, probably on the basis of previous HTGR experience, which shows low fission-product release from coated-particle fuels. As in all HTGR's, the coolant is helium. Similar coolant pressures are used in KSH and Peach Bottom, and coolant temperatures are not widely different. Each of these two reactors is enclosed in a steel pressure vessel, and each has bottom-mounted rod drives. Table 1 is a listing of some important parameters for Peach Bottom and KSH.

Although there is a basic similarity between the reactors in the two plants, the power cycles and equipment for the two are totally different. Figure 4 is

Table 1 Comparison of Peach Bottom and KSH Reactor Data

	Peach Bottom	KSH
Reactor power, MW(t)	115	65
Net electrical output, MW(e)	40	24
Efficiency, %	35	37
Reactor inlet temperature, °F	652	800
Reactor outlet temperature, °F	1318	1355
Reactor coolant pressure, psi	360	360
Average core power density, kW/liter	8.3	6.4
Maximum fuel temperature, °F	2428	2370

Safety

The KSH designers consider a loss-of-coolant accident to be the most important safety consideration. The plant is housed in a reinforced-concrete, industrial-type building that is vented through a filter system and a stack. In the event of a loss-of-coolant accident, the building is pressurized to about 4 psig, returning to near atmospheric pressure in about 15 min. Owing to the large heat capacity and high allowable temperature of the core, cooling need not be established for several minutes after the accident and the accompanying scram. This is a characteristic of all

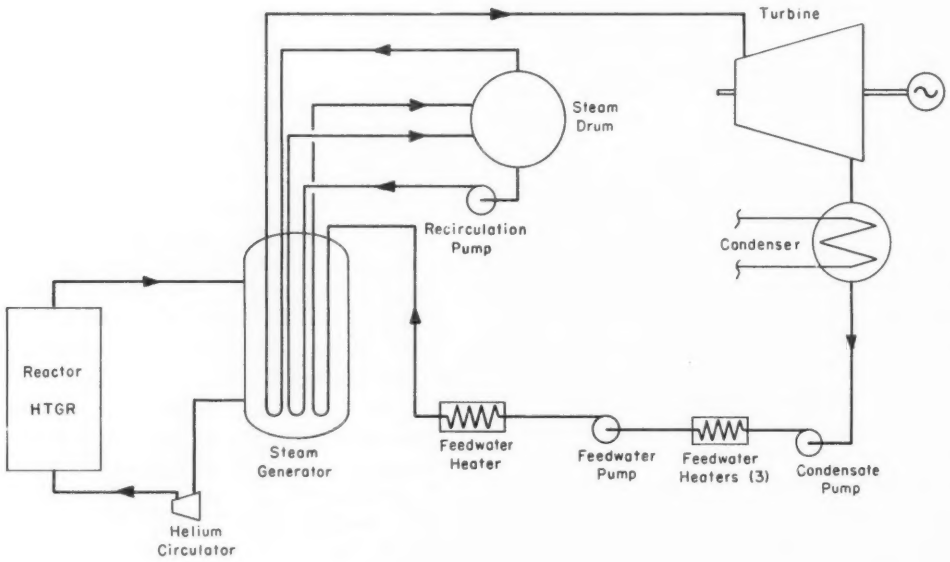

Fig. 4 Peach Bottom power cycle.

a diagram of the Peach Bottom power cycle; this diagram can be compared with the one in Fig. 2 for KSH. Table 2 shows important parameters of the KSH power cycle. The turbine and all three compressor sections for KSH are of the axial-flow type and are mounted on a single shaft. The turbine has 12 stages, and the compressor has 27 stages. The turbine–compressor operates at 8000 rpm, is about 26 ft long, and, because of the low molecular weight of the gas, has a large number of stages for the pressure ratio (2.55). The power-cycle components (reactor, heat exchangers, turbine–compressor) are laid out in a conventional loop arrangement with piping between components.

Table 2 Parameters of the KSH Power Cycle

Helium mass flow, kg/sec	40.6
Turbine inlet temperature, °F	1346
Turbine inlet pressure, psi	367
Turbine outlet temperature, °F	842
Turbine outlet pressure, psi	144
Turbine pressure ratio	2.55
Reactor power, MW(t)	65
Regenerative heat-exchanger duty, MW(t)	74
Precooler inlet temperature, °F	214
Precooler outlet temperature, °F	59
Intercoolers inlet temperature, °F	147/140
Intercoolers outlet temperature, °F	59/59
Cooling-water temperature, °F	50

HTGR's—and a very desirable one at that. Nitrogen is fed into the core to prevent atmospheric oxygen from contacting the graphite, and the emergency core-cooling systems are started within about 20 min.

When the water and steam circuits of the power cycle are eliminated, the safety problems associated with leaks into the reactor coolant system are also eliminated. These problems include the reactivity effect of adding steam to the reactor coolant and the attack of graphite by steam, both of which can occur as a result of a steam-generator tube failure but which have not actually been major safety problems in steam-cycle HTGR plants. The greatest advantage in eliminating the steam generators is probably in the realm of operating reliability, inasmuch as major problems have occurred with steam generators in a number of gas-cooled reactors.

Control

Two basic methods of control are provided in KSH, and they are designed to work in conjunction with each other.[5,6] For most power changes between 50 and 100% of rated power, the inventory of gas in the power circuit will be changed (increased to increase power, and vice versa) by pumping into or out of a storage tank. The reactor power is controlled to maintain a constant outlet temperature. Pressure ratios, temperatures, and thermal efficiency will be almost unchanged over the power range. For rapid load decreases and for operation below 50% power, a gas-bypass system is provided; this system consists of the circuits associated with valves V-1 and V-2 shown in Fig. 2. For load reductions below 50% or for more rapid load drops than can be accommodated by coolant inventory control, valve V-1 opens to bypass gas around the turbine. Valve V-2 opens to bypass some of the gas from valve V-1 (which is much hotter than the turbine exhaust gas) around the regenerative heat exchanger and to maintain constant reactor inlet temperature. Although load decrease by inventory reduction does not result in a significant efficiency loss, the bypass operation is very inefficient. Reactor power required for 50% electrical output is only about 51% if inventory control is used but is 92% if bypass control is used.

FUTURE DEVELOPMENT

The KSH plant represents an important step toward the development of large, gas-cycle, nuclear generating units. Although the construction and operation of the plant will leave a number of important questions unanswered or only partially answered, various other projects under way in Germany and England will provide additional information on the use of a gas-turbine cycle in one form or another.

In Germany, emphasis is on plants using HTGR-type reactors and, looking farther ahead, on gas-cycle plants using gas-cooled fast reactors. In England, attention is focused on the AGR for near-term exploitation and on the HTGR and gas-cooled fast reactors for later construction. The British have performed a number of studies of commercial-size gas-turbine plants with the intent of examining design problems, optimizing economics, estimating costs, and determining areas for future work.

Large, steam-cycle, gas-cooled reactors currently under construction use prestressed-concrete pressure vessels that enclose the entire primary system—reactor, steam generators, and gas circulators. Containment buildings such as those used with PWR's are not provided. One design approach under consideration for gas-cycle plants is to carry the integrated concept forward and enclose the entire power circuit, including the turbine–compressor–generator and the heat exchangers, in a single prestressed-concrete vessel. The other extreme is loop-type construction, with the reactor in a prestressed-concrete vessel and the turbine–compressor–generator and heat exchangers located externally and connected by piping. Examples of these approaches are given in Refs. 7 and 8.

Studies that have been performed indicate that the integrated type of construction is feasible if particular attention is given to the pressure-vessel penetrations to assure their reliability. There are obvious problems in inspection and maintenance of a turbine–compressor–generator unit that is completely enclosed in a prestressed-concrete vessel. One partial solution is to locate the gas-handling portions inside and the generator outside; however, the penetration problems would be accentuated.

Turbine–Compressor Units

Although large turbine–compressor units for a gas-turbine nuclear plant have never been built, there are no fundamental problems in their design and construction. They are expected to be about the same physical size as the largest jet-aircraft engines. Design considerations and requirements have been determined, and some design work has been done, as reported in Refs. 9 and 10.

The basic characteristics of the unit are determined by the gas. Helium machines will have a large number of stages and a low pressure ratio of 2.5 to 3. Allowable blade speeds are high, and high-temperature operation is desirable for maximum cycle efficiency, with 1700°F considered feasible for the turbine.

Reactors utilizing carbon dioxide as the coolant are limited to temperatures well below the normal temperature capability of a gas turbine. Aerodynamic considerations limit the units to lower blade speeds, but fewer stages are required, although pressure ratios will be higher for carbon dioxide than for helium.

Economics

In 1967, Bechtel Corp. performed a study of the cost of constructing and operating a 1000-MW(e) AGR in the United States.[11] A preliminary design was prepared using a hypothetical site, and cost estimates were made on the basis of investor-owned utility ground rules. It was concluded that the cost of energy generated by the station would be competitive with the cost of power generated by commercial nuclear power stations using light-water reactors. A preliminary cost comparison between steam and gas-turbine AGR plants has been made by the British and is reported in Ref. 12. The gas-turbine plant was estimated to have a total capital cost about 10% less than that of the steam-cycle plant, with most of the difference resulting from savings in the cost of the turbine and main process building (turbine hall). Other savings were seen in the heat-transfer equipment and systems, but increased expense in the concrete pressure vessel and electrical equipment partially offset the cost reductions.

Cost estimates for nuclear systems have not proved highly reliable, and caution would be in order in appraising the current economic status of any reactor type. However, it appears that the gas-cooled reactor, especially the HTGR version, may prove to be an economical one—particularly if a gas-turbine cycle is used.

Waste-Heat Utilization

Large quantities of heat are rejected in any thermal-power-generating cycle. With a gas-turbine cycle such as the one used in KSH, the heat is available at temperatures that are high enough (above 200°F) to be useful for desalination or other process applications. With relatively minor modifications of a gas-turbine cycle, the quantity of useful heat available can be increased at a very slight decrease in cycle efficiency and therefore at a low cost. Reference 13 shows three methods of achieving this goal and evaluates the cost of extra heat available by each method. A minimum temperature of 230°F for useful process heat was assumed, and a typical base cycle giving an output of 600 MW(e) was selected. About 200 MW was rejected by the base cycle at 230°F or above, and additional heat (up to 200 MW more) was shown to be obtainable at a cost estimated to be a few cents per million Btu. The most economical way to increase the amount of heat rejected at the 230°F level was to change the pressure ratios of the compression stages while maintaining the total pressure ratio constant. The total amount of heat rejected is not increased much; the primary effect is to increase the temperature at which some of the heat is rejected.

GAS-TURBINE CYCLES FOR FAST BREEDERS

The ultimate development of the gas-cooled reactor type is considered to be the fast breeder, and there is considerable interest in a direct gas-turbine power cycle for this reactor. Another alternative under study by a European group involves the use of a gas-turbine cycle with a sodium-cooled fast breeder.[14] The idea is attractive because carbon dioxide does not react violently with sodium as water does; CO_2 also has good thermodynamic properties at the design temperatures for sodium-cooled fast breeders. For safety reasons, steam-cycle plants always use an intermediate sodium loop between the reactor coolant loop and the steam–water loop. If CO_2 were used as the power-cycle fluid, the intermediate loop would not be needed and the plant would be simplified with no loss in thermal efficiency. Extensive preliminary investigations were planned for 1968–1969 to determine whether the concept would be pursued, but their outcome is not known.

Several gas-cooled fast reactor concepts have been reported.[15-17] As is the case with thermal-neutron reactors, gas-turbine or steam power cycles may be used. Gas temperatures at the core outlets vary widely between these designs, ranging from 1225 to 1800°F, depending on the degree of extrapolation from current technology. Economic calculations tend to agree, however, that the power cycle should be optimized in the direction of low thermal efficiency. For the most efficient cycles, large regenerative heat exchangers are required. The heat-exchanger costs drop rapidly as cycle efficiency decreases, and the heat-exchanger cost decreases are not entirely canceled by increasing costs

in the fuel and reactor areas. For the variety of predicted parameters (core power density, capital and fuel costs, etc.), thermal efficiencies of 30 to 35% were found to be optimum, although values of 40 to 50% can be achieved.

Although water reactors have dominated the American scene, the first- and second-generation gas-cooled reactors have achieved a high degree of development abroad. In their present form, the second-generation reactors (AGR's) may be economically competitive with the light-water reactor systems (keeping in mind that generalized economics comparisons are hazardous at best). However, the prospects for continuing improvements in the AGR type are of more interest than its present status. The third-generation reactors (HTGR's) have operated as small-size units, and larger units are under construction or planned. As a parallel line of development, the gas-turbine cycle is available for use with either the second- or third-generation gas-cooled reactors, giving possible capital-cost savings. Construction of the KSH plant, which will demonstrate the nuclear gas-turbine concept, has begun in Germany, and operation is scheduled for 1973. The gas-turbine power cycle may also be used with fast breeder reactors.

Considering the possibilities for continuing development of gas-turbine nuclear power plants, persons in the United States who are interested in power reactors should keep abreast of progress in Europe especially.

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The Outlook for Nuclear Power-Station Capital Costs

By William Steigelmann*

The availability of sufficient electric energy at low cost is essential to the economic development of any nation. Although other forms of energy are essential as well, the most flexible, most widely used, and most fundamental form of energy in our industrial base is electricity. As is well known, the historic pattern in the United States is for the installed electric generating capacity to be doubled every 10 years. The installed generating capacity of electric-utility organizations in the United States is expected to be approximately 344,000 MW(e) at the end of 1970, 668,000 MW(e) in 1980, and 1,260,000 MW(e) in 1990. Of these totals, nuclear power will provide about 11,000 MW(e) by the end of 1970, 150,000 MW(e) by 1980, and 509,000 MW(e) by 1990 (Ref. 1).

During the past 3 years, the estimated capital costs of a nuclear power plant in the 800- to 900-MW(e) range has increased from about $120/kW to over $200/kW. It is important to note, however, that the time cost has not increased by this range but rather the *estimated* (predicted) cost, since the true cost is a quantity that cannot be known until the plant is completed. Some of the principal reasons for the sharp increase in the predicted cost of nuclear power plants during the past few years are:

1. Escalation of labor and materials costs.
2. Increase in money cost and projected lengthening of construction schedules.
3. Added cost of structures and systems associated with the evolutionary interpretation of safety requirements, including engineering and qualification testing of equipment.
4. Increase in contingency allowances in anticipation of design changes required as a result of interpretations of safety-related analyses made after systems and structures are in an advanced stage of construction.

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The downtrend in orders for nuclear power plants, which resulted in part from the higher estimated costs, began in 1968 and continued into 1969. Orders in 1968 were for 17 units totaling nearly 16,000 MW(e), and the estimated 1969 figures are 10 units totaling about 10,000 MW(e).

Serious delays in construction schedules of many nuclear power plants became evident during the latter part of 1968, along with substantial increases in construction costs. In a number of cases, orders for fossil-fired plants with shorter construction schedules were placed by utilities to meet power requirements that the nuclear plants would have met under original schedules. These orders helped boost total orders for fossil-fueled plants during the year to what appears will be an all-time high of close to 30,000 MW(e).

Public concern about the environmental effects of all types of large steam electric generating stations also has become more intense, and this undoubtedly will make the selection of appropriate sites for all types of power plants even more difficult in the future. Favorable sites for utility power stations are becoming more difficult to obtain owing to the demands of other industries and urban developments. A utility must plan for generating facilities long in advance of the time they are needed to meet system load growth. A major problem will be to find sufficient sites to build the required generating facilities to meet the system demands. Successful planning must consider the types and sizes of generating equipment which will be utilized in the future.

The most significant development in the utility industry in the past 20 years has been the increase in generating capacity and the increase in size of individual units and of generating plants. The "economies of scale" that have been demonstrated in the case of nuclear power stations have helped to accelerate the trend toward unit sizes greater than 1000 MW(e). It is

expected that this trend will continue, but at a slower rate, and that the maximum-size unit may increase to about 3000 MW(e) during the next 20 years. The size of station will undoubtedly be dictated by both economics and site problems, including transportation of large equipment items and concern about possibly adverse effects of the power station on the environment.

It is reasonable to assume that approximately 90% of the nuclear-fueled electric-power generating units that will be ordered in the United States during the next few years will have a rating of at least 800 MW(e) and will be either the boiling-water type or the pressurized-water type. The technical problems associated with the design of such units to assure safe and efficient operation have virtually all been solved, but remaining are a number of problem areas related to finding the optimum way to obtain the lowest total project costs. Virtually every nuclear power project has experienced delays in commercial operation and increased costs over those forecast initially. Through hindsight, it is possible now to see how some of these penalties could have been avoided, usually through a stronger approach to project management, the implementation of more comprehensive quality-assurance programs, or the selection of different personnel and organizations to participate in the project. For whatever the reason—project delays, the need for additional safety-related systems, or rising costs of labor and material—increased capital costs have severely narrowed the economic advantage thought to be enjoyed by nuclear-fueled units in the 1965 to early 1968 time period. In a talk before the Metropolitan New York Section of the American Nuclear Society in October 1969, for example, Louis H. Roddis, who was then vice-chairman, and who is now President, of the Consolidated Edison Company of New York, stated that a 1-year delay could add $8 million to $10 million to the cost of constructing a million-kilowatt nuclear-fueled generating plant.

The AEC requires that each applicant file a statement of financial qualifications and an estimate of costs with its license application.[2] In responding to this requirement, some utility organizations submit cost estimates in accordance with the Federal Power Commission system of accounts, many give only the "total nuclear production plant cost" (this is all that is now required), and some give a breakdown in accordance with their own system. Variations in format are particularly evident in the "indirect cost" category, which includes items such as engineering and design, staff training, and interest during construction.

In order to present a basis for deriving a relation for predicting the cost of a "standard plant," the capital-cost data for 40 nuclear power plants with generating capacities greater than 800 MW(e) have been examined and analyzed (see Table 1). In addition to the total unit cost of a plant as last reported, the table gives an "adjusted" cost for the plant if built under standardized conditions, data on the plant rating, dates of the cost estimate and of expected commercial operation, the reactor vendor, and the distribution of costs across the various accounts. It must be recognized that information on some of these items is rather indefinite or not clearly reported (e.g., ultimate net capability and date of commercial operation).

The "standard plant" to which the adjusted unit costs pertain is a single-unit installation of an investor-owned utility with (1) the same capability rating as the given plant, (2) a location at a typical site that does not require special facilities for heat rejection, and (3) commercial operation scheduled for June 1974. The basis for the cost adjustment is given in Table 2.

In addition to accounting for the rise in the cost of labor and materials, the escalation factor, r, represents the effects of today's higher cost of money and the added cost of hardware and project-management controls found necessary both of themselves and in order to meet continually evolving safety requirements.

An analysis of the adjusted-unit-cost data yields the following relation of total capital cost C, in $/kW(e), as a function of net electric-power capability P, in MW(e), for "standard plant" units larger than 800 MW(e):

$$C = 4.45 \times 10^4 / P^{0.8} \pm 15\%$$

Specific values for the plant unit capital cost at power-plant sizes of usual interest are:

P, MW(e)	C, $/kW(e) (±15%)
800	212
850	202
900	193
1000	178
1100	165
1150	159
1200	153

The 1000-MW(e) plant is included here because many studies and economic evaluations are based on a plant with this rating. It should be noted, however, that no plant of this size is either now under construction or

Table 1 Nuclear Power-Plant Economic Data Summary

Station name (Unit No.)	Owner	Ultimate rating[a] MW(t)	Ultimate rating[a] MW(e)[c]	Reactor vendor	Dates[b] Cost estimate[d]	Dates[b] Operation[e]	Unit capital cost, $/kW(e) Stated[f]	Unit capital cost, $/kW(e) Adjusted[g]	Cost distribution,[h] % Land	S&I	RPE	TGU	AEE	MPPE	DC	E&D	IDC	OIC	IC
Boiling-Water Reactors																			
Shoreham (1)	LILCO	2535	854	GE	7/69	5/75	2/4	205	1.1	20.1	45.0	27.7	5.3	0.8	100[i]	*i*	*i*	*i*	*i*
Brunswick (1 and 2)	CP&L	2550	855	GE	11/69	3/74,76	189	183	0.5	21.0	37.6	22.0	4.9	1.4	87.4[j]	*j*	12.6	*j*	12.6[j]
Cooper (1)	CPPD	2480	810	GE	12/67	4/72	167	200	0.5	11.6	31.5	24.3		7.1	75.0	3.3	18.1	3.6	25.0
Browns Ferry (1 and 2)	TVA	3293	1064	GE	8/66	10/70,71	112	166	0.3	9.7	35.4	26.1	3.6	0.9	76.0	4.6	6.7	12.7	24.0
Browns Ferry (3)	TVA	3293	1064	GE	12/67	10/72	133	165		9.5	37.0	26.4	2.9	0.5	76.3	3.3	7.1	13.3	23.7
Enrico Fermi (2)	Detroit Ed.	3434	1182	GE	4/69	2/74	187	190	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Peach Bottom (2)	*k*	3295	1115	GE	10/67	4/71	148[l]	169	0.1	15.4	34.1	24.2	4.7	1.8	80.3	N.A.	10.7	N.A.	19.7
Peach Bottom (3)	*k*	3295	1115	GE	10/67	4/73	131[l]	147		10.7	35.7	30.0	5.0	0.5	81.9	N.A.	11.4	N.A.	18.1
Indian Point (4)	Con. Ed.	3293	1115	GE	6/69	4/75	197	180	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Indian Point (5)	Con. Ed.	3293	1115	GE	6/69	6/77	184	170	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Pressurized-water Reactors																			
Davis–Besse	*m*	2788	900	B&W	7/69	12/74	200	198	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Oconee (1, 2, and 3)	Duke Pwr.	2568	886	B&W	7/69	5/71,72,73	105	134	0.6	N.A.	N.A.	N.A.	N.A.	N.A.	86.1	5.1	8.3	0.5	13.9
Rancho Seco (1)	SMUD	2452	800	B&W	7/69	5/73	180[l]	181	0.4	15.4	31.1	30.1	5.3	0.7	83.0	5.9	10.0	1.1	17.0
Arkansas Nuclear One (1)	Ark. P&L	2568	880	B&W	10/69	12/72	186	199	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Three Mile Island (1)	Met. Ed.	2535	835	B&W	7/69	9/71	198[l]	210	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Calvert Cliffs (1)	Balt. G&E	2700	865	C-E	5/69	1/73	163[n]	190	1.1	14.4	38.9	27.2	5.7	0.7	88.0[j]	*j*	12.0	*j*	12.0[j]
Calvert Cliffs (2)	Balt. G&E	2700	865	C-E	5/69	1/74	142[n]	191		12.9	38.7	28.2	6.0	0.3	86.1[j]	*j*	13.9	*j*	13.9[j]
Main Yankee (1)	MYAPC	2700	865	C-E	7/68	4/72	166	187	0.4	13.2	34.2	23.1	4.0	0.7	75.6	N.A.	13.6	N.A.	24.4
Millstone Point (2)	*o*	2700	865	C-E	3/69	4/74	209	210	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Joseph M. Farley (1)	Ala. Pwr.	2766	865	W	9/69	12/74	189	204	0.5	17.4	43.9	32.2	5.6	0.4	100[i]	*i*	*i*	*i*	*i*
North Anna (1)	VEPCO	2766	875	W	4/69	3/74	211	209	0.3	14.6	30.8	17.1	3.0	0.4	66.2	3.3	13.6	16.9	33.8
Seabrook (1)	*p*	2774	895	W	4/69	9/74	207	205	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Beaver Valley (1)	*q*	2774	882	W	6/69	6/73	215	224	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Surry (1 and 2)	VEPCO	2456	812	W	3/68	3/71,72	159	200	0.1	12.1	28.1	22.8	3.1	0.9	67.1	3.4	10.7	19.4	32.9
Donald C. Cook (1 and 2)	AEP	3391	1093	W	1/68	6/72,73	132	154	0.4	6.2	41.3	26.3	4.9	0.7	79.8	N.A.	N.A.	N.A.	20.2
Diablo Canyon (1)	PG&E	3391	1103	W	12/68	1/73	139	150	0.2	10.9	32.7	28.8	3.6	1.2	77.4	4.4	N.A.	N.A.	22.6
Diablo Canyon (2)	PG&E	3391	1103	W	6/68	6/74	135	142		8.6	35.4	28.2	4.6	0.5	77.3	3.5	N.A.	N.A.	22.7
Salem (1)	PSE&G	3391	1093	W	4/68	3/72	140	156	0.6	10.9	29.5	23.6	6.4	0.5	71.5	N.A.	12.2	N.A.	28.5
Salem (2)	PSE&G	3391	1093	W	4/68	3/73	115	135		10.4	33.0	27.1	7.2	0.2	77.9	N.A.	10.1	N.A.	22.1
Sequoyah (1 and 2)	TVA	3594	1171	W	10/68	10/73,74	137	151	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	74.6	N.A.	10.7	N.A.	25.4
Trojan (1)	Portland GE	3570	1153	W	7/69	9/74	168[l]	160	0.4	15.7	32.2	26.1	3.0	0.7	78.1	N.A.	10.6	N.A.	21.9
Zion (1)	Comm. Ed.	3391	1115	W	7/68	6/72	147	163	0.9	N.A.	N.A.	N.A.	N.A.	N.A.	80.6	3.4	N.A.	N.A.	19.4
Zion (2)	Comm. Ed.	3391	1115	W	7/68	6/73	135	158		N.A.	N.A.	N.A.	N.A.	N.A.	82.2	1.7	N.A.	N.A.	17.8

N.A. = not available.

[a]For each unit.
[b]Month/year.
[c]Approximate net capability.
[d]Usually date of filing with AEC/DRL. Cost estimate may have been prepared several months earlier.
[e]Expected date of commercial operation.
[f]At time of expected commercial operation.
[g]To July 1974 commercial operation.

[h]Nomenclature used in the FPC system of accounts:
S&I = structures & improvements.
RPE = reactor plant equipment.
TGU = turbogenerator units.
AEE = accessory electrical equipment.
MPPE = miscellaneous power-plant equipment.
DC = direct cost.
E&D = engineering and design.
IDC = interest during construction.
OIC = other indirect costs.
IC = indirect cost.

[i]All indirect costs included in direct cost accounts.
[j]All indirect costs other than IDC included in direct cost accounts.
[k]Phila. Elect. Co., Public Service E&G Co. (N. J.), Atlantic City Elect. Co., and Delmarva Pwr. Co.
[l]Plants with cooling towers.
[m]Toledo Edison Co. and The Cleveland Elect. Ill. Co.
[n]In 1968 dollars, not escalated to startup.
[o]Conn. P&L, Hartford Elect. Light, and Western Mass. Elect.
[p]Public Service of New Hampshire and United Ill.
[q]Duquesne Light Co., Edison Elect. Co., and Penna. Pwr. Co.

Table 2 Basis for Cost Adjustment

Situation	Cost-adjustment factor
Multi-unit plants	
Dual units	Increase by 6%
First of two units	Decrease by 2%
Second of two units	Increase by 10%
Third of three units	Increase by 2%
Geographical location	
Southeastern region (Tenn.–N. C. and below)	Increase by 5%
All other	No change
Heat rejection	
Cooling towers or other special facilities	Subtract $7/kW(e)
Once-through cooling	No change
Escalation to June 1974 completion	Increase by $(1 + r)^t$, where t is the time from announced date of commercial operation to June 1974 and r is the escalation factor

Cost-estimate date	Plant ownership	r
Prior to 1969	Public	0.085
Prior to 1969	Investor	0.065
1969	Public	0.065
1969	Investor	0.050

being offered by a reactor vendor. The result obtained here for an 800-MW(e) plant is in excellent agreement with the value obtained in a survey conducted by the Yankee Atomic Electric Co.: $200/kW(e) for an 825-MW(e) plant entering commercial operation in mid-to-late 1973 (Ref. 3).

The economic competitiveness of a nuclear power plant depends upon much more than its capital cost: the cost of generating a unit quantity of electric energy, mills/kWh, is the only truly pertinent parameter, and the lower fuel cost of nuclear plants enables them to compete favorably with fossil-fueled plants having lower capital and operating costs (for staff and services) but higher fuel costs.

It is interesting to note that there are some indications that there are no overall savings in going beyond an 800–900-MW(e) unit at the present time. The increased construction periods, uncertainties in schedule, need for greater reserve generating capacity, and longer downtimes for refueling and maintenance all combine to produce electric-energy-generation costs that are essentially equal to those achieved with larger-sized units.

Attention should also be focused on the rather strong possibility that we have reached the point where the cost of producing a kilowatt-hour of electricity will no longer decrease with time as it has continuously during the past but will at last increase as the "economies-of-scale" flatten out and inflation catches up with technological advances. This point was made by Francis M. Staszesky, Executive Vice-President of Boston Edison Co., at the Atomic Industrial Forum's third annual Senior Management Forum[4] and by C. Maxwell Stanley, President of Stanley Consultants, Inc., in his keynote address before the 1969 American Power Conference.[5] Milton Shaw, Director of AEC's Division of Reactor Development and Technology mentioned this point in testimony before the Joint Committee on Atomic Energy.[6] Other industry observers, notably James Lane of Oak Ridge National Laboratory, hold to the opposite course, predicting a continual and significant decrease in the cost of electric energy (in constant dollars), including the transmission and distribution areas.[7] Everyone agrees however, that the rate of inflation will be a key determining factor.

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ON THE DEVELOPMENT OF FAST BREEDERS

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INTRODUCTION

The principle of breeding is as old as the development of nuclear reactors. As early as 1944, Fermi and Zinn did their first design work for a fast reactor.[1] From then until the years around 1960, United States, Great Britain, and Russian fast reactors have been developed, which we now classify as the first generation of fast breeders[2-8] (see Table I). EBR-I, EBR-II, EFFBR, DFR, and BR-5 are the most prominent reactors of this first generation. The principal fuel was metal, the sodium temperatures were modest, the cores were small, the breeding external, and all the attention went to the doubling time.[9-10] Economic considerations, however, shifted this attention from doubling time to the fuel cycle.[11] It became apparent that a high burnup is a condition *sine qua non* for a sound fuel cycle and economic attractiveness. The paper of Sampson and Luebke[12] opened the era of the second generation of fast breeders in evaluating the potential and features of fast breeders with PuO_2/UO_2 as fuel. Following that, it was particularly the groups of GE[13] and Karlsruhe[14] that were actively going into this new field. After the Vienna Conference of 1961,[15] this ceramic fast reactor scheme received worldwide attention. At first it was the Doppler coefficient that was the center of interest.[16,17] This subject led to major undertakings, including the SEFOR project of SAEA, AEC, GE, and Karlsruhe together with EURATOM.[18,19] But shortly thereafter most of the attention went to the sodium void coefficient, after having recognized that large ceramic reactors might have a positive, partial sodium void effect.[20] Following the somewhat too-general idea that all power coefficients should be negative, a number of large design studies were initiated in the US.[17,21] To do this, another important parameter had to be fixed: the target size of these reactors. That, in turn, required a first assessment of a possible time scale for development. During 1963/64, worldwide understanding was that fast reactors should and could be economically available by the end of the 1970's. That, in turn, made it clear that 1000 MW(e) is the target size. So the above-mentioned four major studies in the US, namely that of GE, Westinghouse, B&W, and Combustion Engineering, were conducted. They all showed distorted core configurations in order to depress the sodium void effect. Occasionally, this hurts the breeding potential, but emphasis had shifted from breeding to economy.[22] Also, in Great Britain, France, and Germany, such large design studies were conducted; however, all did not show distorted cores. At the 1965 Argonne Conference,[23] the reactors of the existing studies were analyzed as a system and it became apparent that the sodium void comes into action only if a major accident of an incredible nature takes place.[24] The chain of events that cause such an accident were analyzed, and in so doing, the two-phase sodium-flow phenomena, together with the capability of subassembly boxes to withstand pressure peaks, became the center of interest.[23,25-27] But that meant the departure from a so-far-observed route of development: The reactor design should be placed on a consistent and conceivable chain of accidental events, forming the design basis accident; the *ad hoc* postulated maximum (and incredible) accident therefore at that time was no longer the design guidance.[23] Most reactor groups of the world decided then to have an early prototype reactor of something like 200 to 300 MW(e). This was the case for the GE, Westinghouse, and the AI group, for France and for Germany. The British and Russians obviously were ahead, they were already engaged in their PFR and BN-350 prototype reactor designs. The basic idea was, and still is, to

TABLE I

First Generation Fast Breeder Reactors

	USA				USSR			UK	France
	CLEMENTINE	EBR-I	EBR-II	EFFBR	BR-1	BR-2	BR-5	DFR	RAPSODIE
Reactor Power									
Thermal, MW(th)	0.025	1.2	62.5	200	0	0.1	5	72	20
Electrical, MW(e)	0	0.2	20	66	0	0	0	15	0
Core									
Fuel	Pu-Metal	U-Metal	U-Metal	U-Metal	Pu-Metal	Pu-Metal	PuO_2	U-Metal	PuO_2/UO_2
Core volume, liters	2.5	6	65	420	1.7	1.7	17	120	54
Fuel rating av, MW(th)/kg fissile	0.0016	0.02	0.3	0.37	0	0.008	0.1	0.24	0.14
Power density av, MW(th)/liter	0.01	0.17	0.8	0.45	0	0.06	0.3	0.5	0.32
Linear rod power max, W/cm	(av 50)	300	450	250	0	150	200	(av 320)	(av 210)
Neutron flux max, n/(cm^2 sec)	(av 5×10^{12})	1.1×10^{14}	3.7×10^{15}	4.7×10^{15}	5×10^{10}	1×10^{14}	1×10^{15}	2.5×10^{15}	1.8×10^{15}
Primary Heat Transfer System									
Coolant	Hg	NaK	Na	Na	-	Hg	Na	NaK	Na
Coolant temperature									
Core inlet, °C	40	230	370	290	-	30	375	200	410(450)
Core outlet, °C	120	320	470	430	-	60	450(500)	350	500(540)
Coolant mass flow, m^3/h	0.6	80	2200	5500	-	6	240	1800	800
Number of coolant loops	1	1	2	3	-	1	2	24	2
Time Schedule									
Design	1945	1945					1956		1958
Construction	9/1946	1949	1957	8/1956			1957	3/1955	1962
First criticality	11/1946	8/1951	10/1961	8/1963			6/1958	11/1959	1/1967
Full operation	3/1949	12/1951	4/1965	8/1966	1955	1956	7/1959	7/1963	3/1967
Shutdown	6/1953	1963	-	-	1956	1957	-	-	-
Remarks	1. fast reactor, 1. Pu-fueled reactor	1. nuclear electricity generation Pu-Core since 1962	Reactor Plant with integral fuel processing facility	Since 10/1966 out of operation			UC-Core since 1965		(RAPSODIE is not really a reactor of the first generation, it belongs, to a large extent, to the second generation)

have a round of fast breeder prototype reactors that can be compared to the Yankee, Dresden, Indian Point round of thermal prototype reactors in the 1950's. The function of such prototype reactors would be the demonstration of technical feasibility on a commercially significant scale, somewhat in combination with providing a test bed for commercially developed better fuel and reactor components. More recently, these prototype reactors are called demonstration plants by the US groups. The European countries continue to follow that line, whereas the USAEC shows concern about the necessity of excessive testing of reactor components, including fuel and instrumentation, and therefore is considering a more stretched out timing. This is deeply interrelated to the question of safety and the operational availability of sodium-cooled reactors, and the question of which way to go has been heavily debated ever since.

There is no debate that an early prototype requires an intermediate step; more general experience has to be collected before such undertakings can take place. In the US, this step is well covered by the EBR-II, EFFBR, and more specifically and recently by the SEFOR reactor. In Great Britain, this intermediate step is DFR; in France, the recently and so successfully started RAPSODIE reactor fulfills this requirement. Germany is somewhat late, but the KNK reactor of Interatom, which is in its final construction stages at Karlsruhe, is engineering wise in this function—its first core is a thermal but highly compact core, and the second core will be a fast core, leading to the fast, 20-MW(e), KNK-II reactor by 1971. The Japanese recently started a larger fast reactor program and, remarkably enough, among their first project goals is to have a 100-MW(th) fast reactor by 1972 which, in a way, lines up with EBR-II, DFR, and the above-mentioned reactors.

This particular line of experimental reactors gives overall engineering experience.

FAST REACTOR PHYSICS

One of the main reasons for distinguishing between the first and the second fast breeder generation is their physics. As this session does not allow for a special paper on fast reactor physics, some detailed remarks on that field may be reasonable. Fast breeder reactors of the first generation have metallic fuel and are small; therefore, they have a hard spectrum, the lower end of which does not really touch the resonance region. The important contributions to the power coefficient come from fuel thermal expansion. The second generation of fast breeders has a soft spectrum which definitely covers the resonance region (Fig. 1); therefore, the physics of such reactors have to deal in detail with that resonance region, making it the most difficult part of all reactor physics. The first effect to be explored was the Doppler coefficient. The calculation was first done by Goertzel,[28] but the investigation, taken up by Nicholson,[29] still for a somewhat hard spectrum, was later extended and refined by the

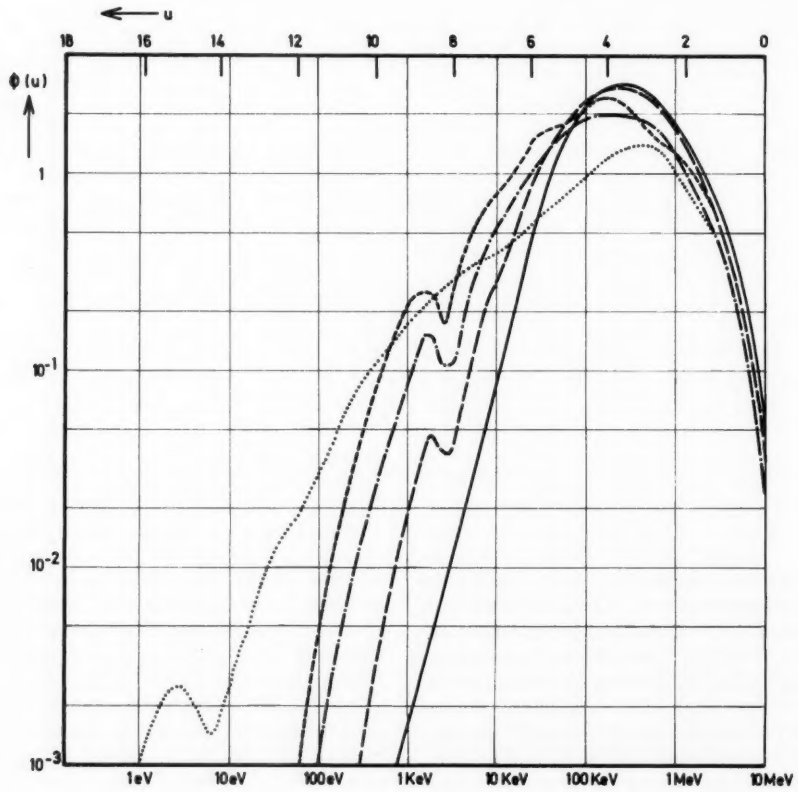

Fig. 1. Neutron flux spectra of fast reactors.

Argonne, GE, Karlsruhe groups, and others.[30-32] Today one can say that the calculation techniques have been established.[33,34] To measure the Doppler coefficient, two principally different approaches are possible. One approach is to measure the Doppler coefficient in action when it terminates a power excursion; according to basic ideas of GE and Karlsruhe, the already-mentioned SEFOR project is designed and executed along these lines.[18,19] The other approach is to measure the Doppler effect in a fast, zero-power critical facility by sample heating; the proper interpretation of that created some difficulties for some time, but contributions of Storrer[35] and others have helped to solve this problem. A great variety of satisfactorily accurate Doppler measurements in criticals now exist. The expected SEFOR results will enlarge the domain of Doppler measurements into higher temperatures and operational conditions, thus leading to a generally satisfactory situation.[36]

The investigation of the sodium-void coefficient was both experimental and theoretical and turned out to be difficult in both areas. The principal tool of exploring the sodium-void coefficient is the fast, zero-power critical facility. Practically all such existing facilities make use of platelets, mostly 2×2 in., a typical thickness being $\frac{1}{8}$ of an inch. In the portion of the spectrum, where $d\phi^+/dE$ governs the physics of this effect, there are large heterogeneities that influence the absolute size of this effect, and large computer programs on the basis of a fairly detailed and cumbersome theoretical analysis are necessary to extrapolate from measured values to values that are significant for a fast power reactor design. It seems that this technique is at hand.[37,38] The theoretical prediction of the sodium-void coefficient has led to the more refined schemes of calculating large fast reactors that have the soft spectrum of the second generation. Particularly for calculating the sodium-void effect, it is necessary to have a two-dimensional code that gives the proper weighing spectrum for group constants in the various parts of the core and the blanket. A very careful preparation of the group constants is then required. It turns out that it is not so much the calculational procedure that creates uncertainties, but the sensitive dependence on microscopic input data and the process of preparing group constants that are of strong influence. Departing from the old idea of having universal sets of group constants,[39,40] there now exist extensive and very detailed programs for preparing a set of group constants for each particular reactor case; i.e., the MC^2 program of ANL,[41] the Galaxy program of the

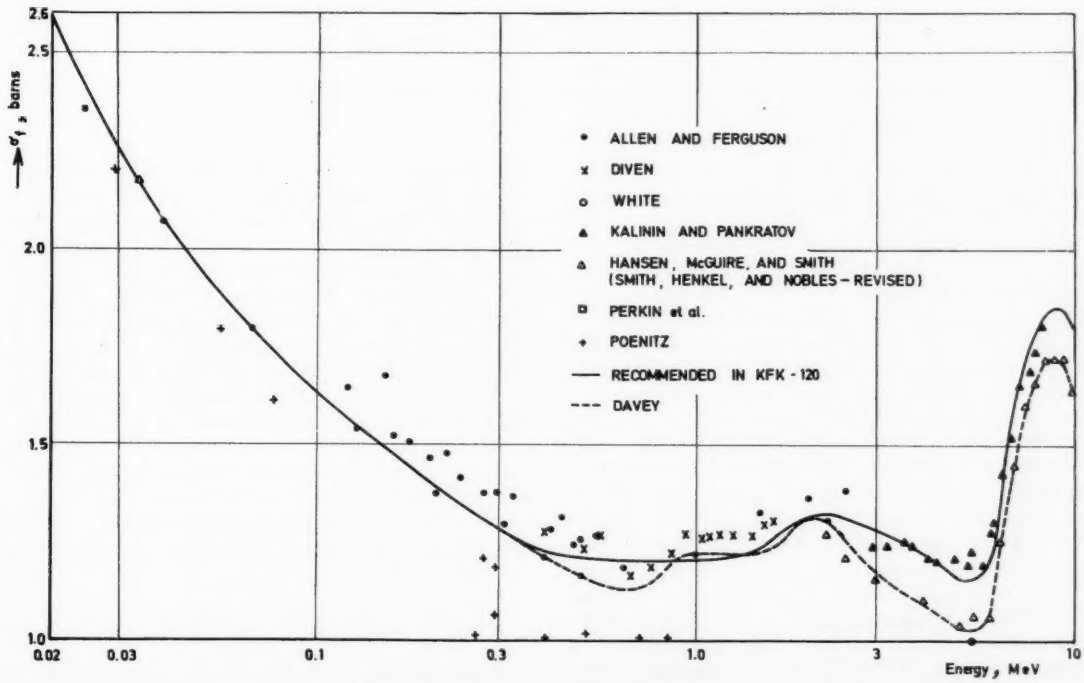

Fig. 2. Fission cross section of ^{235}U.

UKAEA,[42] and the Migros program of Karlsruhe.[43] These programs take into account the self-shielding effect of the prevailing potential scattering cross section of the core, the temperature broadening of the fission and absorption resonances, the influence of scattering resonances, and the influence of weighing spectra. To make all that possible, a tremendous input of microscopic cross-section data is needed. Due to the activities of groups like EANDC,[44] and evaluating and judging compilations of groups like that of Brookhaven[45] and Karlsruhe,[46] the knowledge and availability of these data have improved considerably during the past few years. The most striking event in the area of microscopic input data was the Schomberg measurement[47] of the capture-to-fission α ratio of ^{239}Pu. Schomberg's data would have hurt the breeding ratio by 7 to 9 points and, therefore, the fuel cycle economy by as much as 0.1 mill/kWh. It would have further increased the sodium-void coefficient by 40%, which is due to the sharper raise of ϕ^+ in the respective spectrum areas. Theoretical indications by Pitterle and Barre,[48,49] together with recent measurements of Gwin, seem to indicate that the damage is only half as large. This has been confirmed more recently by a Doppler experiment.[50] Other remarkable trends in this area are the downward trend of the ^{235}U (n,f) standards, and particularly the downward trend of ^{238}U (n,γ) data[51-53] (Figs. 2 through 5).

A remark should be made on the interconnection of extended reactor physics calculations and computer capabilities. The sodium-void calculations, in particular, and the calculation of large, fast, second generation breeders, in general, today require three-dimensional calculations; in the case of the sodium-void effect, for instance, two space dimensions and one energy dimension. This just fits the calculational capability of today's computers, IBM 360/65 (or better 360/91) or CDC 6600. Some years ago all reactor problems were treated only in two dimensions (one space, one energy dimension or one space, one time dimension). As the art develops, it is probable that four dimensions can and will have to be handled and this requires the next generation of big computers. It should be realized how strong this interlink is (Table II). By and large, the reactor situation is satisfactory within certain limits. Further improvements can be expected as microscopic input data become better, computer capabilities increase, and results of large critical plutonium experiments become available. Here also, the situation has improved significantly as large plutonium facilities become operative. The first was ZEBRA in Great Britain (1962), followed by Masurca in France and SNEAK in Germany.

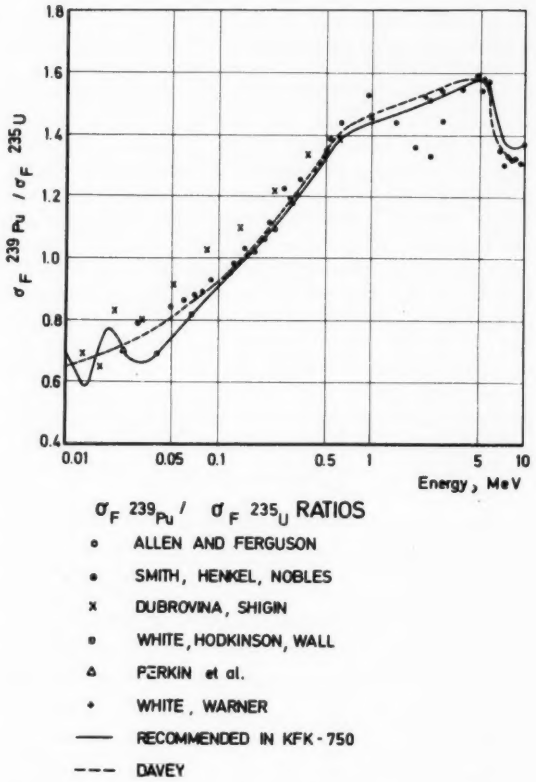

Fig. 3. Ratio of ^{239}Pu and ^{235}U fission cross section.

They became critical on the same night (December 15, 1966), and the ZPPR in the US goes into operation about now. It is almost certain, that with the large plutonium experiments to come, the art of reactor physics will improve further and significantly. Among other things, particularly, will be the verification of group constant sets for certain classes of reactors. Last, but not least, attention should be focused on the Dutch STEK (KRITO) experiment, which is designed to measure the reactivity influence of true fission-product samples.[54] This is important, because both differential and integral data on fission products are missing, and are of increasing importance as longer burnups become realistic. A list of existing zero-energy facilities is attached to this paper (Table III).

THE FUEL ELEMENT

A burnup on the order of 100,000 MWd/ton is mandatory for fast reactor fuel elements to burn something like 100% of the original fissionable

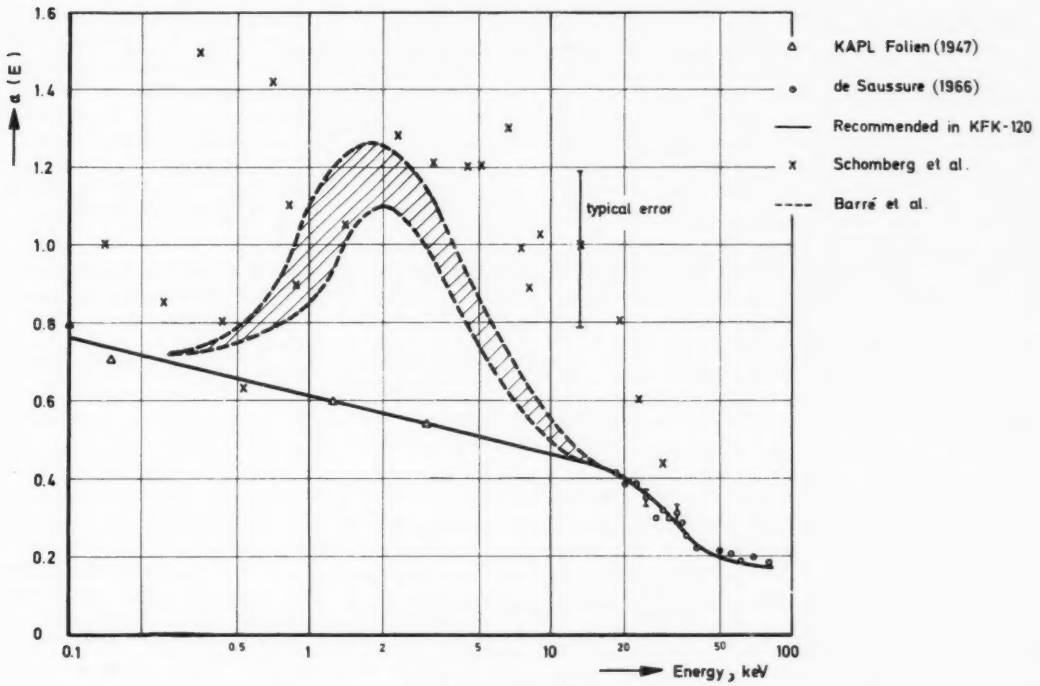

Fig. 4. $\alpha(E)$ of ^{239}Pu.

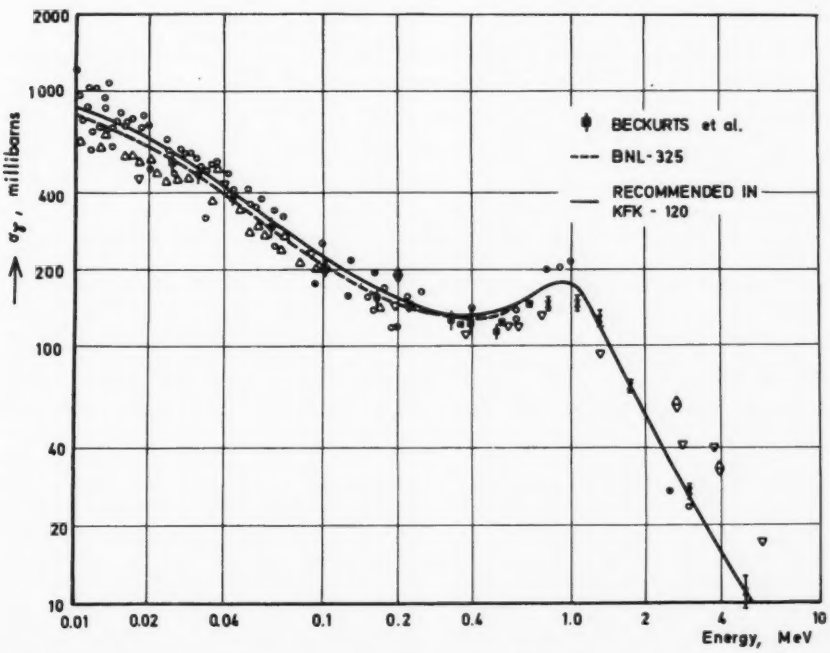

Fig. 5. Capture cross section of ^{238}U.

TABLE II

Fast Reactor Calculations and Computer Capabilities

Generation	Year	Typical Computer	Maximum Number of Dimensions for	
			Parameter Studies	Single Calculations
1	1954-1960	Univac I and II IBM 650	1 (e.g., zero-dimensional multigroup calculations)	2 (one-dimensional multigroup diffusion calculations)
2	1960-1965	IBM 7090 Univac 1107	2 (1-D diffusion calculations)	3 (1-D burnup calculations, 2-D diffusion calculations)
3	1965-1968 from 1968 on	CDC 6600 IBM 360/91 IBM 360/85 CDC 7600	3 (1-D burnup calculations, 2-D diffusion calculations)	4 (3-D diffusion calculations)

TABLE III

Fast Critical Assemblies

	Location	Year First Critical	Short Description	Fissile Material	Typical Core Size, liters (for average reflector thickness)
ZPR-III	Argonne, Idaho	1955	Horizontal split-table machine	^{235}U, ^{239}Pu (600 kg)	600
ECEL	Atomics International, California	1960	Horizontal split-table; thermal driver	^{235}U, 25 kg of ^{233}U were used in some assemblies	100 (test zone)
VERA	Aldermaston, UK	1961	Vertical, split-table	^{235}U, ^{239}Pu (40 kg)	400
BFS	Obninsk, USSR	1961	Vertical, fixed	^{235}U	1800
ZEBRA	Winfrith, UK	1962	Vertical, fixed	^{235}U, ^{239}Pu (400 kg)	3000
ZPR-VI	Argonne, Illinois	1963	Horizontal, split-table	^{235}U	3000
ZPR-IX	Argonne, Illinois	1964	Horizontal, split-table	^{235}U	3000
FRO	Studsvik, Sweden	1964	Vertical, split-table	^{235}U	65
MASURCA	Cadarache, France	1966	Vertical, fixed	^{235}U, ^{239}Pu (200 kg)	3000
SNEAK	Karlsruhe, F.R. of Germany	1966	Vertical, fixed	^{235}U, ^{239}Pu (200 kg)	3000
FCA	Tokai-mura, Japan	1967	Horizontal, split-table	^{235}U, ^{239}Pu planned	3000
ZPPR	Argonne, Idaho	1968	Horizontal, split-table	^{235}U, about 3000 kg of ^{239}Pu planned	3000

content, a condition to make the fuel cycle economic.[22] With metal fuel elements being capable of burnups on the order of only 10,000 MWD/ton, a principle switch to another concept typical for the second generation of fast breeders was necessary. UO_2/PuO_2 fuel is another such fuel concept, provided it has a design that allows for high fission gas release. Low density and high fuel operating temperatures are the principal steps to achieve that.[13,55] This is different from a UO_2/PuO_2, or only UO_2, fuel design for a thermal reactor, where high densities and low fission gas release are required. The intended high fission gas release of fast reactor fuel requires a plenum above or below the axial blanket for collecting the released fission gases. This, in turn, requires the fission gases to travel over long axial distances. The cladding of such a fast reactor fuel

pin must be designed to be free standing and strong enough to withstand the eventually large internal pressure, which might be as high as 100 atm. It is very remarkable that, at present, all fast reactor groups are virtually in agreement on the specifications of such a fast reactor fuel pin. They are roughly as follows: 450 W/cm maximum linear rod power, a pin diameter of 6 mm being consistent with a fuel rating of 1 MW(th)/kg fissile, an active core length of 100 cm, the cladding a 16/13 stainless steel of 0.35-mm wall thickness, and as the most salient point, a smear density in the neighborhood of 80%. In the past, there was some debate about whether or not to provide for axial restraining of the fuel. The APDA group always favored this concept of axial restraining to avoid any reactivity movement; many other groups don't count on, but allow for axial fuel creeping and/or expansion. There is strong indication that fuel under high neutron fluxes and high temperatures is creeping much more readily and easily than under the same mechanical and temperature conditions out of pile,[56] thus enhancing the idea of not restraining the fuel axially and, therefore. avoiding radial swelling. In EBR-II, the present loading with test oxide pins of a type as described above is 105 pins; a similar number having passed the reactor earlier after having reached a burnup of 6% of heavy atoms.[57] According to published data,[58] a somewhat smaller, but still significant number of such oxide pins of the British project have passed the DFR, having reached even higher burnups. At present there is a Franco-German-Benelux subassembly of 77 pins in the DFR aiming for 50,000 MWd/ton, and in parallel, 7 + 19 pins are, or respectively will be, irradiated in the Belgian reactor BR-2 under adjusted fast flux conditions. At present, this is the basis of fuel experience for the above-mentioned round of 300-MW(e) fast oxide breeder prototypes. This basis, though sufficient, is rather narrow and not satisfactory. Broader fuel test experience is among the missing and most urgent developmental requirements.

The cladding material considered for the fuel of these early prototypes is, in all cases, a 16/13 CrNi stainless-steel version. It seems to be impossible to pass the 700°C mark with the hot spot, midwall, maximum cladding temperature. At present, this limits the sodium outlet temperature as a main factor. Extended irradiation tests with cladding material specimens have been executed by practically all fast reactor groups. In the early years one was only concerned about displacement damages. At that time, the completely new phenomenon of (n,α) reaction with the related helium bubble formation changed the picture, and, in 1963, the phenomenon of high-temperature embrittlement came up.[59,60] More recently, the stainless-steel swelling due to void collapsing beyond doses of 10^{22} n/cm^2 and at high fast neutron fluxes came into the picture.[61] Today one finds the following situation: Doses of 10^{22} n/cm^2 have been obtained at three different places—DFR, EBR-II, and BR-2—doses beyond that and up to 10^{23} have been obtained only at DFR, and partially at EBR-II (Fig. 6). According to these results, the applicable tangential creep rupture strengths under irradiation are 40 to 50% lower than without irradiation, the associated tangentials strains there being between 0.3 and 1.5% (Fig. 7).[62] The influence of the radiation dose on the ductility, i.e., the possible tangential strains, is unclear. Particular concern exists on the influence of multiaxial stress configuration to the picture of radiation damage to the ductility as compared with that of uniaxial stress. Therefore, for present fuel pin designs, one has to design for creep strains as low as 0.5 to 1%. The influence of the recently detected stainless-steel swelling on the pin, subassembly, and core design is still unclear. Single DFR data exist that indicate a swelling as high as 10% at doses of 10^{23} n/cm^2 and higher. This phenomenon not only depends on the neutron dose but also on the neutron flux, as it is the flux that creates a supersaturation of point defects; the temperature also influences the phenomenon. Figure 8 illustrates the present picture of this void collapsing.[63] The swelling of stainless steel, if confirmed, would be an awkward, not fully understood phenomenon. A great deal of attention will probably be given to this during the next one or two years. In light of the above remarks, the following conclusion is of great importance: All existing fast-neutron test reactors are of low flux. The reactors in operation have a flux as low as $\simeq 2 \times 10^{15}$ n/(cm^2 sec); the EFFBR [200 MW(th), core A] would be higher by a factor of 2.5, but it is not in operation. The required neutron flux of the 300-MW(e) prototype reactors is at 8×10^{15}, four times as high as the presently available neutron flux (Table IV). With respect to the fuel and the required burnups during fuel pin performance tests, it is possible to artificially enrich the UO_2 of the UO_2/PuO_2 mixture and therefore to compensate for the lower fluxes by higher overall microscopic fission cross sections. But with respect to the applicable cladding neutron doses, no such trick is possible. The bad thing is that the missing factor 4 might be decisive as one realizes that void collapsing appears to a reasonable extent only beyond 10^{22} n/cm^2, not to mention the dependence of this phenomenon on the flux level. One can further speculate that at even higher doses and fluxes, still other phenomena might occur—UFO's—"Unforseen Flux Obstruc-

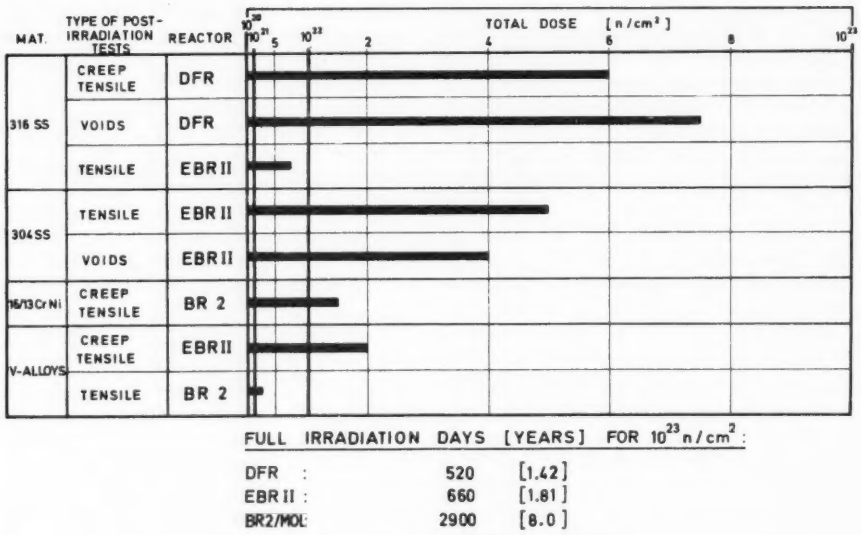

Fig. 6. Obtained doses of cladding irradiations.

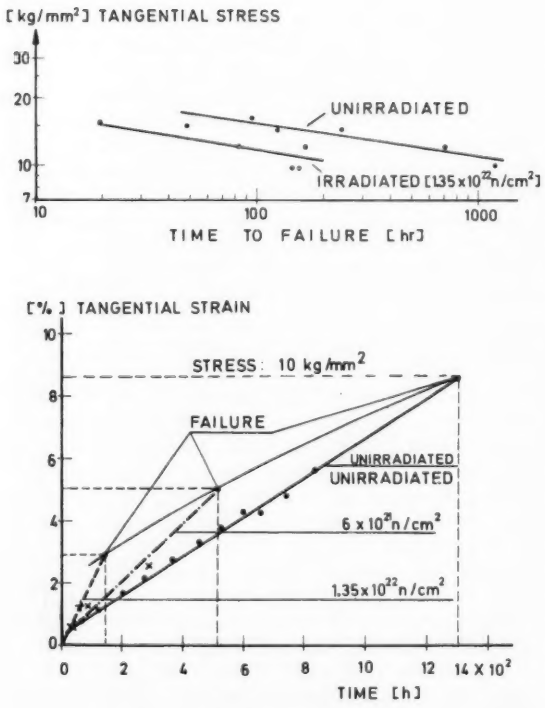

Fig. 7. Effect of irradiation on the creep and stress-rupture strength of pressurized 316L tubes at 650°C [from I.P. Bell et al.[62]].

tions." It seems to be possible, however, to stretch the existing possibilities for irradiation tests and to verify the present approach for having a 16/13 stainless steel as cladding material for the envisaged 300-MW(e) early prototypes. This would be particularly so if the EFFBR could again come into operation. But independent of building the early 300-MW(e) prototypes, it is now very clear that a fast-neutron, high-flux materials test reactor is a necessity. This test reactor must have a flux of 1.5×10^{16} n/(cm^2 sec) or more to have a certain flux margin for the present-day fuel and, therefore, to allow for shorter irradiation periods consistent with a considered dose rate. Looking at the possible time schedule of such a test reactor, one realizes that such a fast MTR cannot go into operation before 1975. On that schedule, not only the cladding doses and today's fuel pins have to be tested, but also the high-performance fuels, particularly the carbides. Irradiation experience with the carbides, to date, is not very encouraging, but there is indication that the hot fuel concept with linear rod powers of such a carbide pin of 1250 to 1500 W/cm will lead to acceptable swelling rates. The higher breeding ratio of a carbide-fueled core is well known. But it is important that besides doubling time, the fuel rating alone is of great significance as it determines the first core requirements. Up to the year 2000, the first core requirements of fast breeders will largely influence the overall natural-uranium consumption of a compound con-

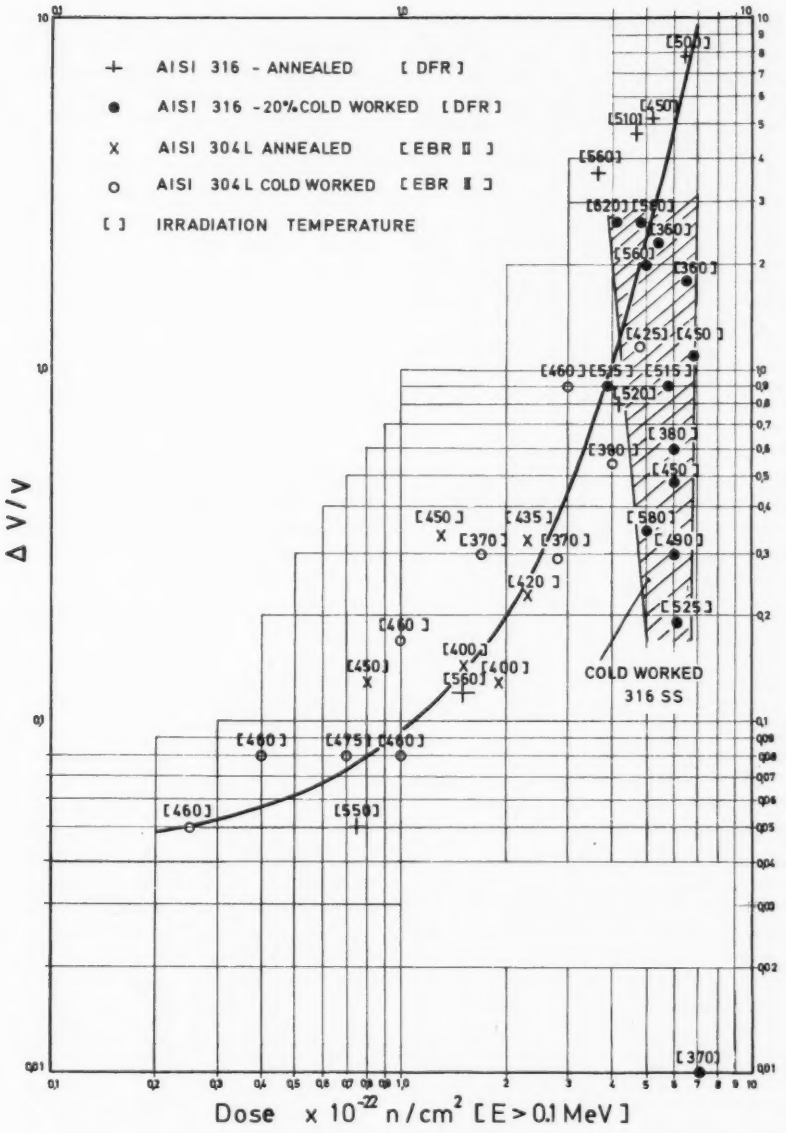
+ AISI 316 - ANNEALED [DFR]
● AISI 316 -20%COLD WORKED [DFR]
X AISI 304L ANNEALED [EBR II]
o AISI 304L COLD WORKED [EBR II]
[ ] IRRADIATION TEMPERATURE
COLD WORKED
316 SS
Δ V/V
Dose x 10^-22 n/cm^2 [E > 0.1 MeV]

Fig. 8. Swelling of stainless steel.

verter/breeder economy.[64,65] Also, the far-reaching question of the required and properly chosen capacity for isotope separation plants is interconnected to that. Therefore, high-performance fuel with a rating of at least 2 MW(th)/kg fissile will undoubtedly come into the picture, thus leading to flux levels of 1.5×10^{16} n/(cm^2 sec). A modern fast-neutron test reactor should not have a lower flux.

FAST REACTOR SAFETY

Fast reactor safety has been a subject for exploration since the beginning. Originally it was the short neutron lifetime that caused concern. It is now clear, however, that the short neutron lifetime is an advantage, provided the instantaneous power coefficient is negative. In that case, the first power peak is terminated within a short

TABLE IV

Neutron Flux of Fast Reactors

	Existing Reactors				Projected		Prototypes	
	EFFBR Core A	EBR-II	DFR	RAPSODIE	RAPSODIE FORTISSIMO	EFFBR Oxide-Core	PHENIX PFR	GFK-Na2
Reactor-Power [MW(th)]	200	42	60	22	40	430	530	730
Total Neutron Flux [$\times 10^{15}$ n/(cm^2 sec)]	4.73	2.07	2.5	1.8	3.0	8.2	7.0	8.7
Neutron Flux above 0.11 MeV [$\times 10^{15}$ n/(cm^2 séc)]	3.51	1.77	2.22	1.6		5.0	4.1	4.8

time scale. One recalls that this time scale is given by $(l/a)^{1/2}$, if l is the neutron lifetime and a is the ramp rate. Also, the inserted energy in that peak is smaller if the neutron lifetime is smaller,[66] as this energy under the first peak is proportional to $(l \cdot a)^{1/2}$. The second concern for fast reactor safety stemmed from the EBR-I meltdown accident.[67] A partial, but instantaneous power coefficient (the bowing effect of fuel elements due to thermal gradients) was positive there. From that time on, attention was focused on the power coefficients. The phenomenon of the bowing effect became transparent,[68] but other coefficients came in with the advent of the second generation of fast breeders. As mentioned before, it was, at first, the Doppler coefficient. The above-mentioned measurements in fast criticals, the elaborate calculations of that effect, and the SEFOR program to come, clarify the size and sign of that Doppler coefficient. The role of the Doppler effect in an accidental sequence of events has not always been entirely clear. It is twofold: First, it terminates the first power peak of a fast excursion and decreases the energy that can be pumped into it and, therefore, gives time to the shutoff system to react. By the same token it helps to establish inherent operational stability; second, the Doppler coefficient also strongly influences the energy release figures of a Bethe Tait calculation, and makes these Bethe Tait results less sensitive against the data of the equation-of-state of the involved reactor fuel.[69,70] This became fully apparent between 1962 and 1965. Discussion of the partially positive sodium-void coefficient started in 1963 and is still going on as far as its role in an accidental sequence of events is concerned. As mentioned before, such voiding of the inner core zones, which have positive sodium-void contributions, really has to take place. For that, either initiating reactivity ramp rates are necessary, which can happen only if there is a complete failure of the shutoff system, and it is debatable whether this is a reasonable assumption, or, one has to experience the blockage of a subassembly from the coolant flow, and this can lead to sodium ejection from this one subassembly. If the sodium ejection is preceded by superheat of the fluid, this ejection is so violent that propagation of this subassembly failure to other subassemblies takes place.[25] In these cases, and only in these cases, the sodium-void coefficient is of significance.[24] Depressing the sodium-void coefficient by distorting the cores, e.g., to make them extremely flat to enhance leakage, hurts the breeding capability considerably and the question comes up whether it is reasonable to depress breeding through all the operation of a fast breeder because of accidents that are considered unrealistic. The situation is further complicated because the Bethe Tait codes, which predict the accidental energy release, are essentially codes that start from a homogeneous core model. But the ejection of sodium from cooling channels, which acts as kind of a ramp rate multiplication (or initiation, respectively), makes reference to the pin-cooling channel geometry and is therefore a strong feature of heterogeneity. Because of this, it is difficult to predict the energy release of a Bethe Tait event with confidence. Finally, one has to realize that the after-meltdown decay heat of large 1000-MW(e) reactors is very large [10 to 100 MW(th)], and one has to take it away from a configuration that has experienced such a hypothetical accident. This requires active engineered safeguard measures. If one goes into detail, one realizes that active, and therefore engineered, safeguard measures are indeed unavoidable. If this is so, one should concentrate on avoiding sodium ejection by engineered safeguard measures, such as instru-

mentation of each subassembly in the core, diversification of the rods of the shutoff system, a second and completely independent and different shutoff system, properly designed cooling channels and pin surfaces to avoid superheat, and proper design to avoid damage propagation from subassembly to subassembly. I refrain from elaborating a consistent scheme for such safety measures because that would take too much time and would be outside the scope of this paper. But I make the point that the present trend is to put aside the sodium-void effect because one has to rely on engineered safeguards anyway. More generally, one can conclude that the new art of quality control, reliability control, and the probabilistic assessment of failure in a network of possible events[71] has to be built up much more than it has been in the past, although it is clear that, in so doing, one has to establish a certain equilibrium with the previous safety evaluations. I am personally convinced that reactor designers can learn a lot from missile designers in this area.

HEAVY SODIUM COMPONENTS

Mastering the sodium technology requires, first of all, large and reliable sodium pumps. Present sodium reactors have pumps up to 3000 m^3/h ($\simeq$12,000 gpm). The mechanical pump type is now prevailing; EM pumps are used today only for special purposes. Most 300-MW(e) prototype reactors will have pump sizes of 5000 m^3/h ($\simeq$ 20,000 gpm), but present studies for pumps go far higher. There is general agreement that extended pump tests are required.

In addition to the sodium pumps, the steam generator specifically requires special attention. Large sodium-component test rigs have to be brought up to develop and test these engineering components. The large 35-MW(th) sodium-component test installation at Santa Susanna came into operation in 1965; in the UK, the test rig for sodium pumps came into operation in 1964/65, and these rigs provided the first significantly large test experience. Larger test rigs are now under construction in the US, Russia, France, The Netherlands, and in Germany. It is with high confidence that one can expect the test results necessary for the commitment of 300-MW(e) prototype construction. Table V lists the more important and prototype-oriented test rigs. All the existing fast breeder designs provide a primary and a secondary sodium circuit and the steam-generating turbine circuit as a third circuit. Eliminating the intermediate circuit, because of the related capital cost burden, had been discussed, and it is only recently that Belgonucléaire of Belgium proposed a CO_2 gas-turbine circuit as a second circuit, thus eliminating the intermediate sodium circuit.[72] This scheme is feasible because CO_2 gas turbines do not require such high temperatures for thermal efficiency; rather, high gas pressure is required. This scheme also promises lower capital costs because of the elimination of the intermediate sodium circuit and the much smaller size of a CO_2 gas turbine, if compared with the corresponding steam turbine. In the framework of the German-Benelux fast breeder project, a study is going on to see if such a CO_2 circuit of limited size should be added to their prototype.

OTHER COOLANTS

Second generation fast reactors are large and therefore of low enrichment, they make use of plutonium instead of uranium, which further lowers the fraction of fissile atoms in the fuel, and they make use of UO_2/PuO_2 (or other ceramics) as fuel, such fuel having only half the density of the former metallic fuel. This gives cooling densities[14] of only 300 to 500 kW(th)/liter core, and such cooling densities allow for coolants other than sodium. Helium and steam were then considered as coolants.[73-78] Detailed studies at Karlsruhe and GE, and more recently an ENEA study for comparing sodium, steam, and helium as coolants, have been executed. It is now a fairly well established fact that steam cooling has the same economic potential as the sodium-oxide-fueled fast breeder now under development at many places. However, there is one heavy setback: The high external coolant pressure of steam requires the fuel element scheme of a partially collapsed cladding using either Incoloy-800 or the 12RX72 Swedish steel (or a similar material). For satisfactory testing such a fuel element with collapsed cladding, dry steam, and high external pressure, a fuel test bed is required, or in other words, a small experimental reactor that plays the role for steam that has been played by EBR-II, DFR, RAPSODIE, and KNK for sodium. The GE proposal for an experimental steam-cooled fast ceramic reactor (ESCR) was exactly this kind of a reactor. In Germany, the intended remodeling of the HDR reactor (Heissdampf reactor of AEG at Grosswelzheim, Germany) into a coupled, fast, thermal STR reactor was meant to have the same function. Neither project materialized, for financial reasons and timing. An ESCR or STR would have become operative not earlier, or not much earlier, than the 300-MW(e) sodium prototypes. Steam-cooled fast breeders would be late, therefore, instead of being earlier than sodium-cooled fast breeders. The breeding potential of steam-

TABLE V

Heavy Sodium Component Test Facilities

	Facility	Purpose	Technical Data	Time Schedule
USA	35-MW Sodium Component Test Installation-SCTI	Testing of different steam generators and intermediate heat exchanger	Na-Na-steam system Na:max 650°C (700°C) Steam: 560°C/170 atm	1965 preoperational test 1966 operation
	Sodium Pump Test Facility-SPTF	Testing of pumps	Pump capacity up to 32 000 m^3/h, temp max 650°C	1969 construction 1971 operation
USSR	3-MW Sodium Test Loop	Investigation of steam generator and intermediate heat exchanger models		1960 operation
	Sodium Pump Test Facility	Testing of BN-350 pumps		1966 construction
UK	Sodium Pump Test Facility	Testing of sodium pumps	Pump capacity 1620 m^3/h	1964/65 operation
France	5-MW Grand Quevilly	Investigation of steam generator and intermediate heat exchanger models	Na-NaK-steam system Na: max 600°C (625°C) Steam: 545°C (565°C)/130 atm	1964 operation
	50-MW EDF Test Facility	Testing of steam generator	Na-steam system Na: max 650°C	1967 construction 1969 operation
Germany Benelux	5-MW INTERATOM Test Facility	Investigation of special aspects of steam generators	Na-Na-steam system Na: max 560°C Steam: 500-540°C/200 atm	1963 construction 1965 operation for KNK 1969 operation for SNR
	INTERATOM Sodium Pump Test Facility	Testing of pumps	Pump capacity 5000 m^3/h (15 000 m^3/h)	1967 construction 1969 operation
Japan	50-MW NERATOOM Sodium Component Test Facility	Testing of 50-MW steam generator, 70-MW intermediate heat exchanger	Na-Na-steam system Na: max 650°C Steam: 600°C/215 atm	1968 construction 1970 operation
	2-MW Sodium Test Facility	Investigation of various characteristics of sodium components	Na-system, max 650°C	1969 operation

cooled breeders is limited and there is no foreseeable potential to go to a high performance version, as is the case with the carbide fuel for sodium-cooled reactors. These features are not detrimental if the steam-cooled breeder comes early. They become real concerns, however, if the timing of steam-cooled breeders is delayed. Technically, the gas-cooled breeder is in the same position. There, also, high external coolant gas pressures are applied to the fuel, and undoubtedly a large-scale fuel development with the associated tests either in a fast-neutron material test reactor or in an EBR-II, ESCR-type reactor with gas as coolant, is mandatory. This necessarily results in a different time scale; therefore, it seems impossible that gas-cooled fast breeders could be commercially available before 1985. But the gas-cooled fast breeder has two features that make it attractive even with such a time scale: the potential for high gain breeding, the potential for very high temperatures with the associated direct gas turbine cycle and, related to that, the potential for very low energy production costs coming from both the low capital costs and the low fuel cycle costs. This makes the scheme attractive in spite of the fact this reactor belongs to the *third* and not to the second generation of fast breeders.

THE PRESENT FAST BREEDER REACTOR PROJECTS

At present, the following sodium fast breeder projects are in existence:

1. In the USSR there is the BN-350 prototype reactor at the Caspian sea in the advanced stages of construction. This reactor is designed for 150 MW(e) and for 200 MW(e) equivalent for seawater desalination. The reactor is expected to be ready by 1969.

2. In Great Britain there is the PFR, a 250-MW(e) fast breeder prototype reactor, which is supposed to be ready by 1971.

TABLE VI

Second Generation Fast Breeder Reactors

	USA			USSR	UK	France	Germany
	GE	Westing-house	AI	BN-350	PFR	PHENIX	SNR
Reactor Power							
Thermal, MW(th)	750	540	1250	1000	600(670)	600	730
Electrical, MW(e)	310	212	500	350	250(275)	250	300
Core (reference)							
Fuel	PuO_2/UO_2	Carbide	PuO_2/UO_2	PuO_2/UO_2 or UO_2	PuO_2/UO_2	PuO_2/UO_2	PuO_2/UO_2
Core volume	2000	1000	3000	1900	1320	1150	1750
Fuel rating av, MW(th)/kg fissile	0.82	0.9	1.0	0.96	0.7	0.8	0.76
Power density av, MW(th)/liter	0.31	0.5	0.37	0.5	0.4	0.5	0.37
Linear rod power max, W/cm	500	1000	500	470	450	430	420
Breeding ratio	1.2	1.45	1.3	1.5	1.2	1.1-1.3	1.33
Burnup, MWd/ton	100 000	100 000	75 000		70 000	~50 000	55 000
Primary Heat Transfer System							
Type	Pool	Loop	Loop	Loop	Pool	Pool	Loop
Coolant	Na	Na	Na	Na	Na	Na	Na
Number of coolant loops	3	1	3	5	3	3	3
Pump capacity, m^3/h	5000	16 000	12 000	3200	5000	4800	4670
Coolant temperature							
Core inlet, °C	425	410	425	300	400-420	405(420)	380
Core outlet, °C	590	540	570	500	560-585	565(590)	560
Steam System							
Steam generator							
Number		1	3	5	3	3	9
Type		Pool	Modular			Modular	Pool
Materials: Evaporator			Ferritic		Ferritic	Ferritic	Ferritic
Materials: Superheater + Reheater			Austenitic		Austenitic	Austinitic	Ferritic
Steam conditions							
Temperature, °C	510	480	480	435	510-540	540	505
Pressure, atm	170	170	170	50	162	163	165
Date of Operation	1975	1975	1975	1969	1971	1973	1974-75

3. In France there is the PHENIX reactor, a 250-MW(e) fast breeder prototype reactor, which will be ready by 1973.

4. In the US, a study is going on by GE, together with the ESADA group, on a 310-MW(e) prototype plant; this reactor might go into operation by 1975. Westinghouse is conducting a similar study, also with utilities as partners; the contemplated size is 212 MW(e) and the reactor might also go into operation by 1975.

   Atomics International, with the GPU group, is considering a 500-MW(e) plant; the time scale is similar to that of GE and Westinghouse.

5. Germany, together with Belgium, the Netherlands, and Luxemburg, is designing a 300-MW(e) SNR prototype plant. Construction

TABLE VII

Second Generation Experimental Fast Reactors

	USA		USSR	Germany	Italy	Japan
	SEFOR	FFTF	BOR-60	KNK-II	PEC	JEFR
Reactor Power						
Thermal, MW(th)	20	400	60	58	130	92
Electrical, MW(e)	0	0	12	20	0	0
Core						
Fuel	PuO_2/UO_2	PuO_2/UO_2	PuO_2/UO_2 or UO_2		UO_2	PuO_2-UO_2
Core volume, liter	500	950	53		420	240
Fuel rating av, MW(th)/kg fissile	0.06	0.7			0.34	0.4
Power density av, MW(th)/liter	0.04	0.45	1		0.28	0.35
Linear rod power max, W/cm	650	500	590	420	400	430
Neutron flux max, n/(cm^2 sec)	6×10^{14}	7×10^{15}			4×10^{15}	4×10^{15}
Primary Heat Transfer System						
Type	Loop	Loop		Loop	Loop	Loop
Coolant	Na	Na	Na	Na	Na	Na
Number of coolant loops	1		2	2	2	1
Pump capacity, m^3/h	1360		600	670	1300	2400
Coolant temperature						
Core inlet, °C	370	260	360-450	410	375	370
Core outlet, °C	430	420	600	560	525	500
Date of Operation	1968/69	1973	1969	1971		1972

will start in 1970; 1974 is the date of completion. Germany's share is 70%, that of Belgium and the Netherlands, 15% each.

6. Japan is planning a 200- to 300-MW(e) prototype; it is contemplated to have its start-up around 1976.

In addition to these prototype reactor projects, the following projects are in progress:

1. The SEFOR project: SEFOR will have its start-up in a few months; it is a 20-MW(th) experimental reactor of SAEA, AEC, GE, and Karlsruhe, together with EURATOM.
2. The FFTF project of the USAEC: This will be a 400-MW(th) test reactor; the expected neutron flux is 7×10^{15} n/(cm^2 sec); 5 big loops are envisaged.
3. Italy decided to build the PEC reactor, a 130-MW(th) test reactor; the expected neutron flux is 3.8×10^{15} n/(cm^2 sec); 3 test loops are provided.
4. Interatom of Germany will convert the KNK reactor into the KNK-II reactor, a fast reactor with 20 MW(e), 60 MW(th). 1971 is the expected date for operating this KNK-II reactor; the thermal KNK reactor will go into operation next year.
5. The BOR reactor of Russia: This is a test-reactor with 60 MW(th), which, in a way, is an extension of the BR-5 reactor line; its start-up date is 1969.
6. A 100-MW(th) experimental fast reactor is under design in Japan. It is expected to go into operation by 1972.

A more elaborate table with much more data is attached (see Tables VI and VII).[7,8,79-85]

CONCLUDING REMARKS

Fast breeders have a strong and short range economic incentive. This is particularly so because the present generation of thermal power reactors produce large amounts of plutonium that can be used meaningfully only in fast breeders. They have the long-range potential of breeding and therefore really make use of the existing uranium resources, and more than that, of the ever-increasing vast amounts of depleted uranium.[86,87] Beyond the year 2000, breeding will be a necessity and one should bear in mind there are only 30 years left, i.e., the life span of only one power station. Therefore, there is no doubt that fast breeders are the ultimate solution to the

problem of providing nuclear energy. The international fast reactor community is well under way to accomplish their tasks.

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SYSTEM AND COMPONENT DESIGN FOR FAST REACTORS

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INTRODUCTION

A discussion of system and component design for fast reactors must, of necessity, be selective and limited if it is to conform to reasonable limitations of time and space. Consequently, I will limit this discussion to liquid-metal-cooled fast breeder reactors (LMFBR) and gas-cooled fast reactors (GCFR). More specifically, I will concentrate on sodium-cooled and helium-cooled fast breeder reactors. Steam-cooled fast reactors will not be considered because of their relatively poor potential breeding capability and the apparent decrease in interest in this system. (It should be noted, however, that while much of the information relevant to the GCFR is also applicable to the steam-cooled system, there are difficult materials compatibility problems and control problems that are unique to the steam system.)

It appears that sodium or helium possesses adequate heat transfer capability[1] to meet the operational requirements of fast breeder reactors. Their respective coolant properties are quite different, however, and these differences have a significant influence on the design of the systems and components for the LMFBR and the GCFR. Comparison of the pertinent properties of these coolants emphasizes this potential impact on system and component design.

The heat transfer properties of sodium are superior to helium, but these advantages are partially offset by the need for an intermediate sodium heat transfer system. The total pumping power for the two sodium systems is comparable to a single helium system. Similarly, the total temperature difference between the fuel cladding temperature and steam temperature is comparable because the higher sodium heat transfer coefficient is offset by the additional temperature drop taken in the intermediate heat exchanger. Also, although the breeding ratio of the GCFR is higher than the LMFBR, this advantage is offset by the lower power density and larger critical mass of the GCFR. This results in a longer doubling time for the GCFR. Table I gives a simple comparison of these pertinent characteristics taken from two reactor studies. These studies were not optimized on the same basis, nor do they necessarily reflect comparable levels of design conservatism, but the figures do indicate significant comparability.

The other properties of these coolants are somewhat more difficult to compare, but they can be characterized as follows. Sodium becomes highly radioactive and its chemical activity requires that it be completely isolated from the atmosphere and from water. In addition, it is opaque and a relatively poor lubricant for operation of mechanisms. On the other hand, helium is relatively inert and transparent, but must be pressurized to achieve acceptable heat transfer performance. The relatively high density of sodium combined with its excellent heat transfer

TABLE I

Comparative Performance Characteristics
LMFBR and GCFR Systems

(U-Pu oxide fuel—stainless-steel clad)

	LMFBR[3]	GCFR[4]
Coolant	Sodium	Helium
Coolant pressure, atm	1	85
Coolant outlet temp, °F	1150	1175
Coolant temp rise, °F	340	530
Max cladding temp, °F	1300	1300
Steam temp, °F	950	1000
Breeding ratio	1.40	1.51
Doubling time, years	7.75	9.0

properties provide excellent natural-convection characteristics for removal of fission product decay heat. By comparison, helium provides very poor natural-convection heat-removal capability and forced convection is required to remove fission product decay heat. While both sodium and helium have favorable neutronic properties for use as fast reactor coolants, sodium has a much larger potential reactivity effect. In large reactors, the sodium coefficient of reactivity can be reasonably large and positive and is an important safety consideration.[2]

These basic characteristics of the two fast reactor systems influence the design of the systems and components and may play a major role in the selection of a system for commercial use. As a result, the operability and reliability of these systems and components may, in the final analysis, determine the selection.

SYSTEM ARRANGEMENT

Power operation of sodium- or helium-cooled systems appears to involve relatively straightforward engineering development and extrapolation of existing experience. Because of the very high power density of these reactors, shutdown cooling to reliably remove fission product decay heat imposes more stringent requirements than for the relatively low power density thermal reactors. As a result, many of the system features of fast reactors are imposed by shutdown cooling considerations.

Sodium-cooled reactors can be arranged to provide reliable shutdown cooling by natural circulation with redundant heat removal capability to extract heat from the primary sodium. To achieve reliability of such decay-heat removal systems, loss of coolant from the primary system must be prevented. This has led to rather wide interest in the pot concept (Fig. 1) which places all primary components and interconnecting sodium piping within a single vessel and thereby avoids the possibility of coolant loss due to pipe failure. Similarly, loop systems (Fig. 2) have been designed to avoid loss of coolant in the event of pipe failure. The elevation of the primary loops and all components in them are above the core, which precludes draining the sodium level in the system below the core by a leak anywhere in the system. These features provide extremely high reliability of fission product decay-heat removal.

The need to prevent loss of coolant is also essential in helium-cooled fast reactors. Rapid depressurization, as might result from catastrophic failure of a vessel or pipe in a GCFR system, would result in inadequate heat removal and resultant serious fuel damage. Gulf General Atomic (GGA) has developed a totally contained helium-cooled reactor concept to preclude such failures (Figs. 3 and 4). This concept is now feasible because of the technological advances that have been made in the design and construction of Prestressed Concrete Reactor Vessels (PCRV). It is believed that a major catastrophic-type failure in a PCRV is precluded by the redundancy and inspectability provided by the prestressed cable arrangement. Since all the primary system components are contained within the PCRV, and all the internal volume is pressurized to operating pressure, it is concluded that this system design precludes catastrophic depressurization.

SHUTDOWN COOLING

The arrangement of the LMFBR and GCFR systems reflects the need to minimize the possibility of loss of coolant and to assure reliable fission product decay-heat removal. Shutdown cooling systems are required to meet these requirements. The requirements for a shutdown cooling system can be separated into two basic functions: (1) to remove the heat from the fuel effectively and reliably and at temperatures acceptable to the fuel, structure, and shutdown cooling system; and (2) to transfer the heat to a heat sink and dissipate it reliably under all foreseeable conditions.

In the LMFBR, the fission product decay heat can be removed easily from the fuel by natural circulation of the sodium coolant. It is essential, of course, that the primary system be arranged to provide good natural-circulation flow characteristics with adequate thermal driving head. This can be achieved quite reasonably. For EBR-II (Fig. 5), it is accomplished whether or not heat is removed in the intermediate heat exchanger; i.e., the heat exchanger elevation is sufficiently high in the system relative to the core to provide an adequate cold leg. However, it is essential that natural circulation be established effectively and quickly under all conceivable reactor shutdown conditions. To achieve this, it is necessary to employ up-flow through the reactor during power operation to avoid the necessity of flow reversal to initiate thermal-convection circulation. Similarly, natural convection can be provided reliably in a loop system (Fig. 2) by virtue of the relative elevation of the heat exchanger and core.

In the GCFR, forced-convection circulation of the helium coolant is necessary to remove fission product decay heat from the fuel. The necessary reliability and redundancy requires that the primary circulators, as well as auxiliary circulators, have independent sources of power and control to avoid any common source of failure.

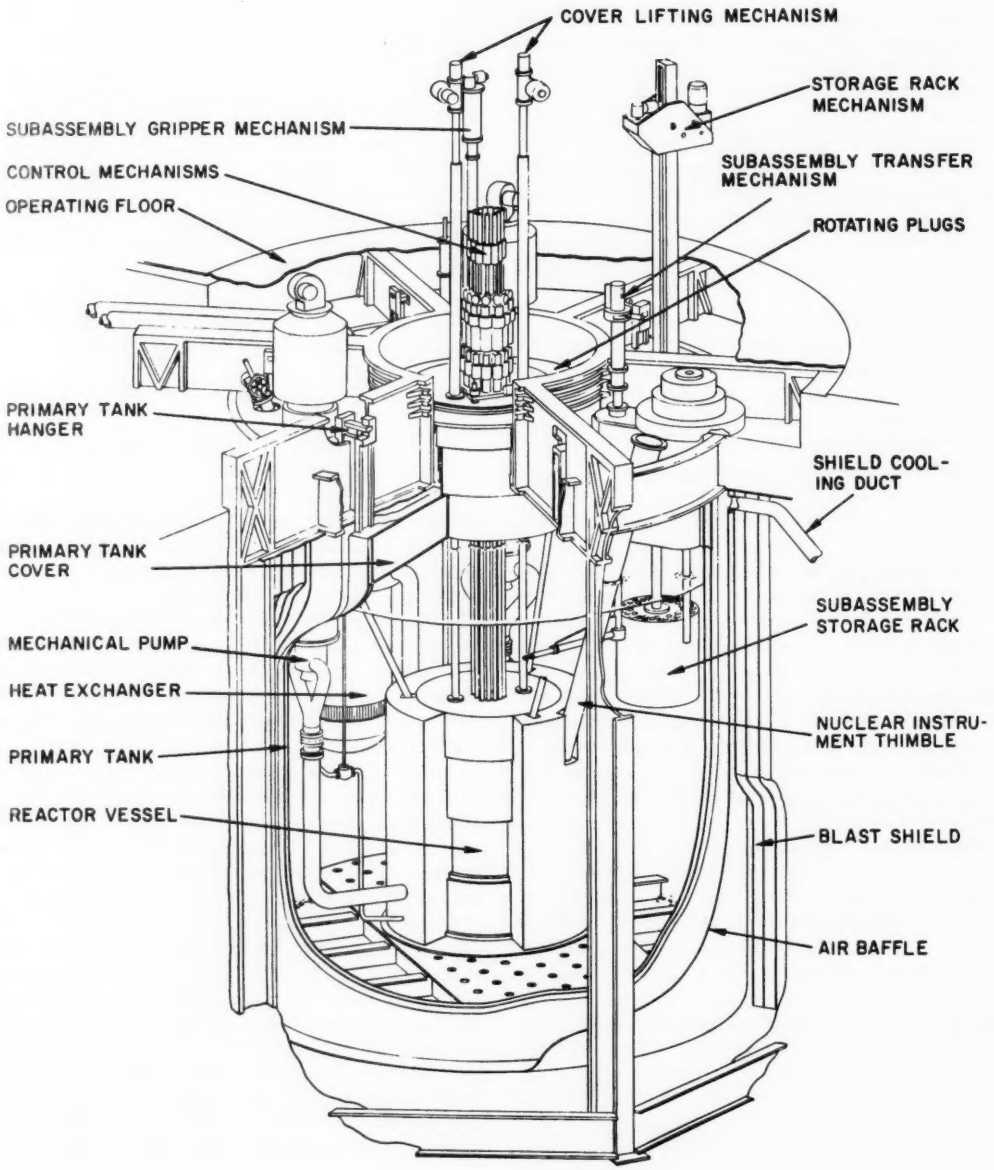

Fig. 1. EBR-II, primary system (ANL photo).

In addition, it is necessary to ensure that adequate forced circulation will continue through the period of depressurization corresponding to the maximum coolant leakage rate resulting from failure of the PCRV. Gulf General Atomic has established what they believe to be maximum leakage rates that might result from failure of a PCRV. Although the drop in pressure is quite rapid (to one atmosphere in less than five minutes), adequate circulation is provided by steam-driven circulators. This results because, even under the most severe conditions of depressurization, the pumping load on the circulators decreases more rapidly (as a result of reduced loading due to depressur-

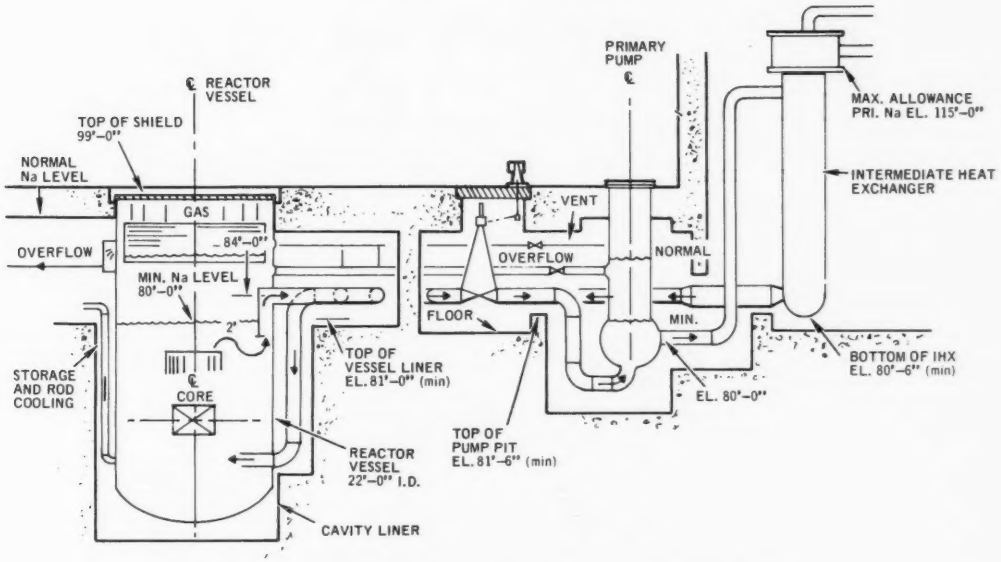

Fig. 2. 1000-MW(e) LMFBR—loop arrangement (AI photo).

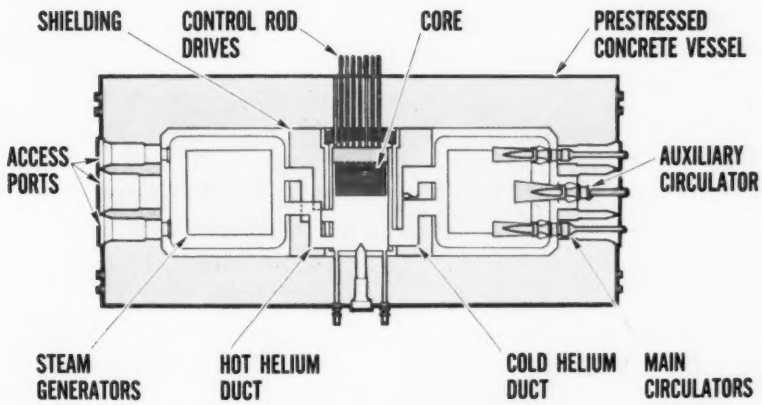

Fig. 3. 1000-MW(e) GCFR steam supply system (GGA photo).

ization) than the steam driving force available to the circulators. Consequently, as long as the steam supply continues and the circulators are operative, it is only necessary to effect overspeed control of the main circulators. Auxiliary, independent circulators are also provided but they must also have a reliable source(s) of power to ensure fission product decay-heat removal from the fuel. In the event that all methods of forced helium circulation fail, the PCRV can be filled with poisoned water, thus flooding the reactor and removing fission product decay heat by natural circulation. Reliable methods of introducing this cooling water into the system under all conditions of reactor temperature (above and below operating temperature) and pressure (atmospheric to operating pressure) will be required.

In both the LMFBR and GCFR systems, the fission product decay heat can be transferred to and dissipated through the steam and condenser cooling water systems to the atmosphere. In fact, the systems can be arranged to do so by natural

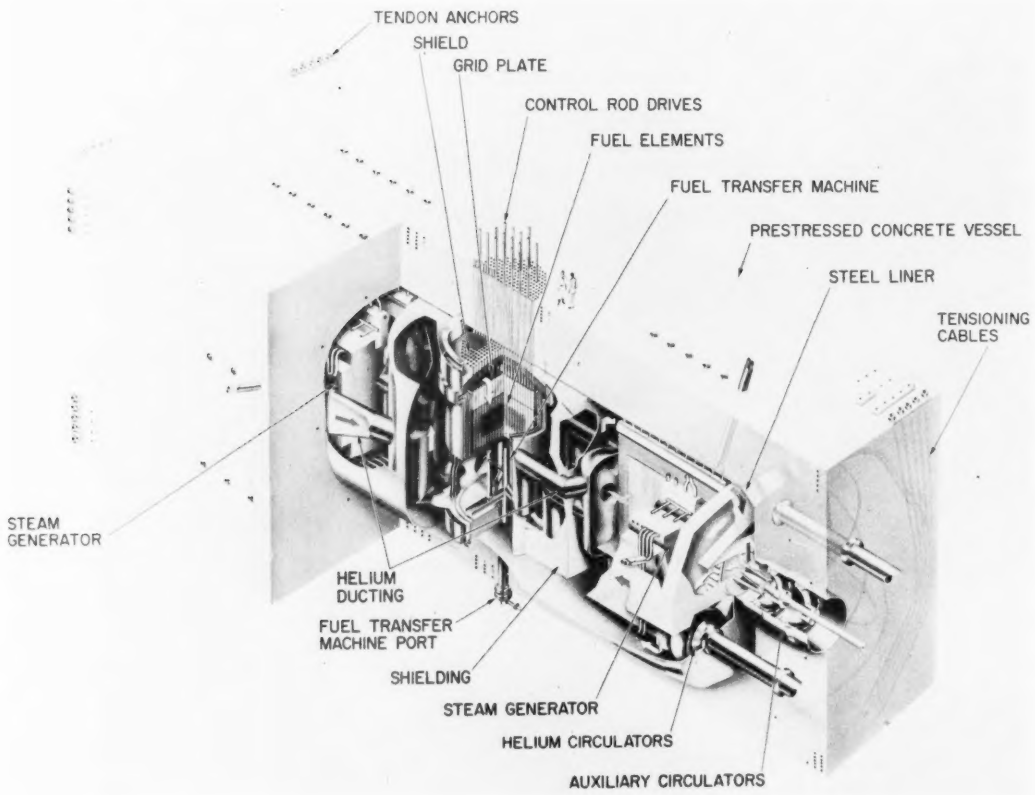

Fig. 4. 1000-MW(e) GCFR steam supply system (GGA photo).

circulation. For example, in EBR-II, fission product decay heat is transferred by natural circulation of the primary and secondary sodium to the steam system. In the event that all auxiliary power is lost (or the system fails to operate due to equipment failure), the fission product heat is automatically dissipated to the atmosphere through two shutdown coolers (Fig. 6). These remove heat from the primary tank by natural circulation of NaK and natural circulation of air. The two systems are always operating at a very low heat-removal rate and require only the opening of dampers in the chimney to achieve full capacity. The dampers are held closed by electromagnets and automatically open in the event of power failure (as well as in response to specific automatic or manually actuated signals).

The GCFR utilizes the normal power system to transfer and dissipate fission product decay heat to the atmosphere. In addition, the heat can be transferred to the PCRV water-cooling system (and utilize part of the large heat capacity of the PCRV structure). The transfer of fission product decay heat by either of these methods requires power to achieve forced circulation of the helium coolant and other heat transfer fluids. If the system is flooded with water to remove the heat from the fuel, transfer of heat to the atmosphere should be relatively easy since the tremendous heat capacity of the water and PCRV would allow considerable time to actuate systems.

FUEL HANDLING

Fuel handling in fast power reactors is more difficult and more demanding than in thermal power reactors because of mechanical problems resulting from the close-packed geometry of the fuel assemblies and thermal problems resulting from the large quantity of fission product decay heat to be removed from the fuel. The fuel handling system may be one of the most critical in the fast reactor plant because: (1) It must perform many difficult and complex operations, the

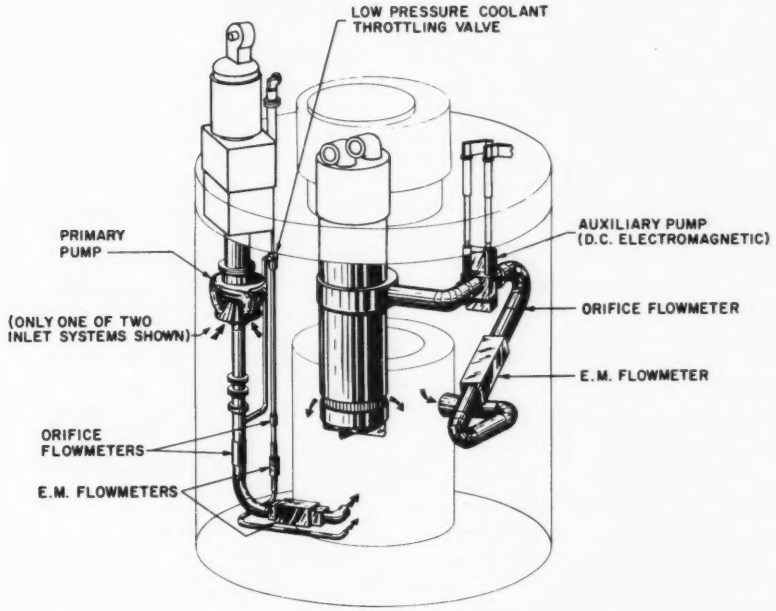

Fig. 5. EBR-II primary piping and component arrangement (ANL photo).

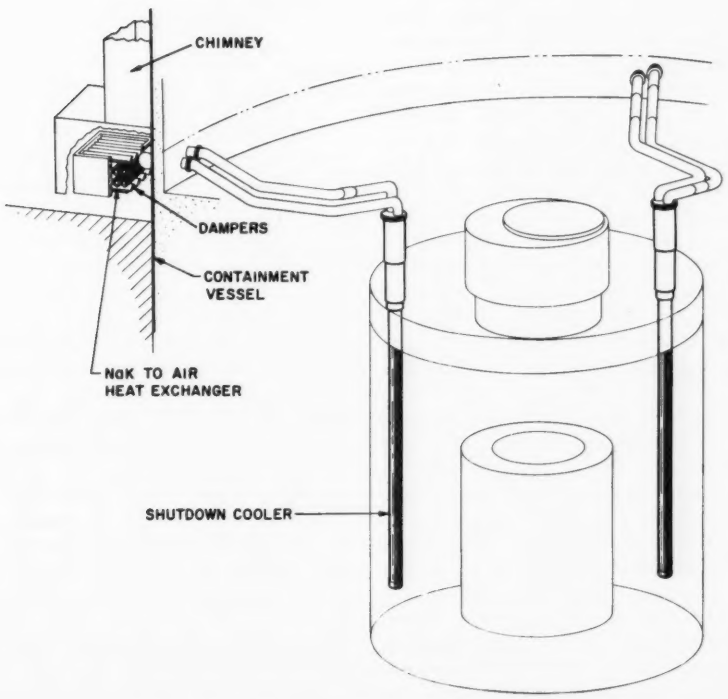

Fig. 6. EBR-II shutdown cooling system (ANL photo).

failure of almost any of which can cause very serious difficulties; and (2) it must perform many of these operations with the reactor shut down, thus contributing to loss of plant factor. Therefore, there are strong economic incentives to perform fuel handling operations efficiently to minimize reactor downtime. This can be accomplished by reducing the frequency of performing fuel handling operations and by reducing the time required to perform the operations.

Fuel handling concepts and procedures are influenced very strongly by frequency and quantity of fuel change. LMFBR fuel handling systems have been characterized on the basis of their frequency of use.[5] For purposes of comparison, "frequent operation" was defined as twice a month, and "occasional operation" as twice a year.

The EBR-II fuel handling system (Fig. 7) is a frequent-operation system and is also referred to as an under-the-plug system. This type of system is characterized by relatively short preparation time to initiate fuel handling operations after reactor shutdown. It requires a minimum of change in reactor system operating conditions to effect the transition from the reactor operating condition to the fuel handling condition. After the reactor has been shut down and the primary sodium pumps have been stopped, the control-rod drives are disconnected from the rods and the reactor cover is raised (as shown in Fig. 7). These operations require approximately two hours after reactor shutdown. The subassemblies are then transferred from the reactor to the storage rack, and exchanged for new subassemblies one at a time. These operations require approximately one hour per exchange. They are performed with the subassembly completely submerged in sodium as shown, and with the sodium maintained at ~700°F—the normal inlet sodium temperature during power operation. After all the fuel handling has been performed, the primary system is restored to the reactor operating condition (reactor cover lowered, control drives connected, etc.), which requires about an hour.

Subassemblies are removed from the primary tank through the fuel unloading machine while the reactor is operating and, therefore, the time involved in these operations does not affect plant factor. The subassemblies are transferred in the interbuilding fuel transfer coffin through the equipment air lock as shown in Fig. 8. (The reactor is in the operating condition and an interbuilding fuel transfer coffin is being moved through the equipment air lock—without violating building containment.)

The occasional systems, on the other hand, are characterized by relatively long preparation time with significant changes in reactor system conditions. This may include a major change in the environmental conditions of the reactor system, such as cooling the system to relatively low temperature, and may also require major physical change in the system, such as opening the reactor vessel for access. The hot-cell fuel handling concept for LMFBR systems is an example of an occasional-operation system (Fig. 9). This system involves opening the primary sodium vessel to the refueling cell. This exposes a large area of the free sodium surface in the vessel to the cell. Cooling the sodium and purging the cover gas prior to this operation should reduce substantially the transfer of radioactivity to the cell. The various operations involved in fuel handling are shown schematically in Fig. 10 and are as follows:

1. Place new fuel in thimbles in fuel storage tank
2. Remove shield plug
3. Place new fuel and thimbles in temporary storage positions at periphery of reactor
4. Remove used fuel from core and place in empty thimbles at periphery of reactor (keeping under sodium during transfer)
5. Remove new fuel from thimble in temporary storage position and place in core
6. Transport thimble and used fuel (with sodium in thimble for cooling) to decay tank
7. Replace shield plug
8. Remove thimble and decayed fuel from storage tank to shipment preparation vault.

Note that steps 1 and 8 can be performed with the reactor operating and do not influence plant factor. Note also that steps 2 and 7 must be accompanied by relatively significant changes in system conditions relative to power operation and that, in total, these steps will require several days.

A somewhat intermediate fuel handling system employs under-the-plug-type transfer operations and a fuel storage vessel external to the reactor vessel. The sequence of operations with this system is described pictorially in Fig. 11. A cross section through the reactor (Fig. 12) shows the in-vessel spent-fuel storage positions within the reactor vessel. As might be expected, the preparation time for this system lies in between the other two systems, with preparation and restoration steps estimated to require approximately one day.

The GCFR requires gradual depressurizing of the system and gradual reduction in coolant flow

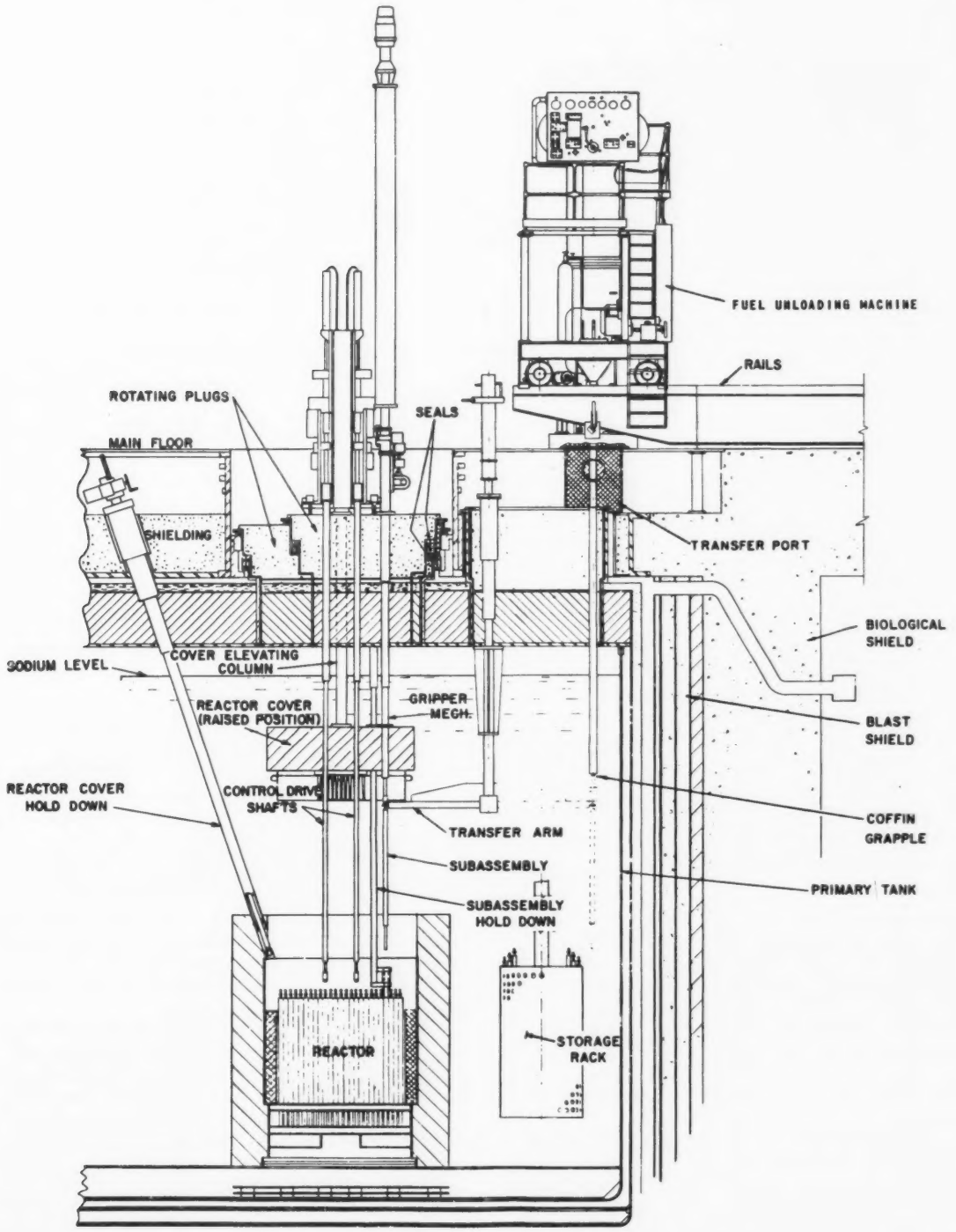

Fig. 7. EBR-II fuel handling system (ANL photo).

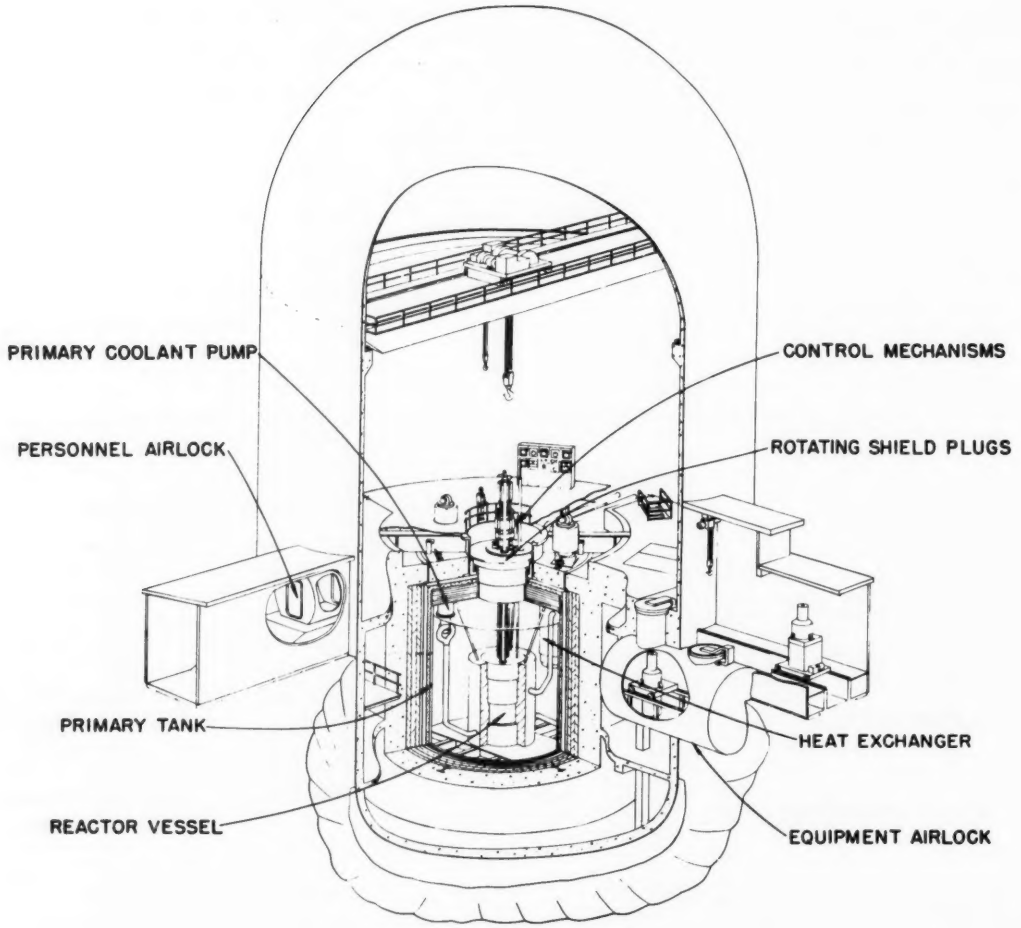

Fig. 8. EBR-II reactor building (ANL photo).

after reactor shutdown. Simultaneously, the system must be cooled to near room temperature to simplify fuel handling and transfer of fuel assemblies to a water-filled storage pool. Although very little effort has been applied to developing a fuel handling system for this reactor (Fig. 13), it would appear that several days might be involved in preparation for fuel handling operations and thus characterize it as an occasional-operation system.

Other features of the design also suggest a "slow but simple" fuel handling system for the GCFR. Each fuel element (Fig. 14) is individually supported in the reactor (Fig. 4) by the lifting rod which penetrates the top shield. This arrangement permits manual vertical movement of the fuel element between the grid plate and the fuel transfer machine (Fig. 15). This type of semi-manual operation tends to be slow, but offers considerable flexibility in instrumenting the fuel elements and considerable latitude in performing the fuel handling operations.

With the increased emphasis on high-burnup fuel, the need for frequent fuel handling has decreased. Systems that operate as infrequently as every year or two can be considered. Operating experience with EBR-II indicates that fission product release from failed or defective fuel elements to the coolant may have to be tolerated. Small quantities of fission products have been released to the coolant and it has been very time consuming to locate and remove the leaky assembly. This process would be prohibitively time consuming with a slow fuel handling system. With

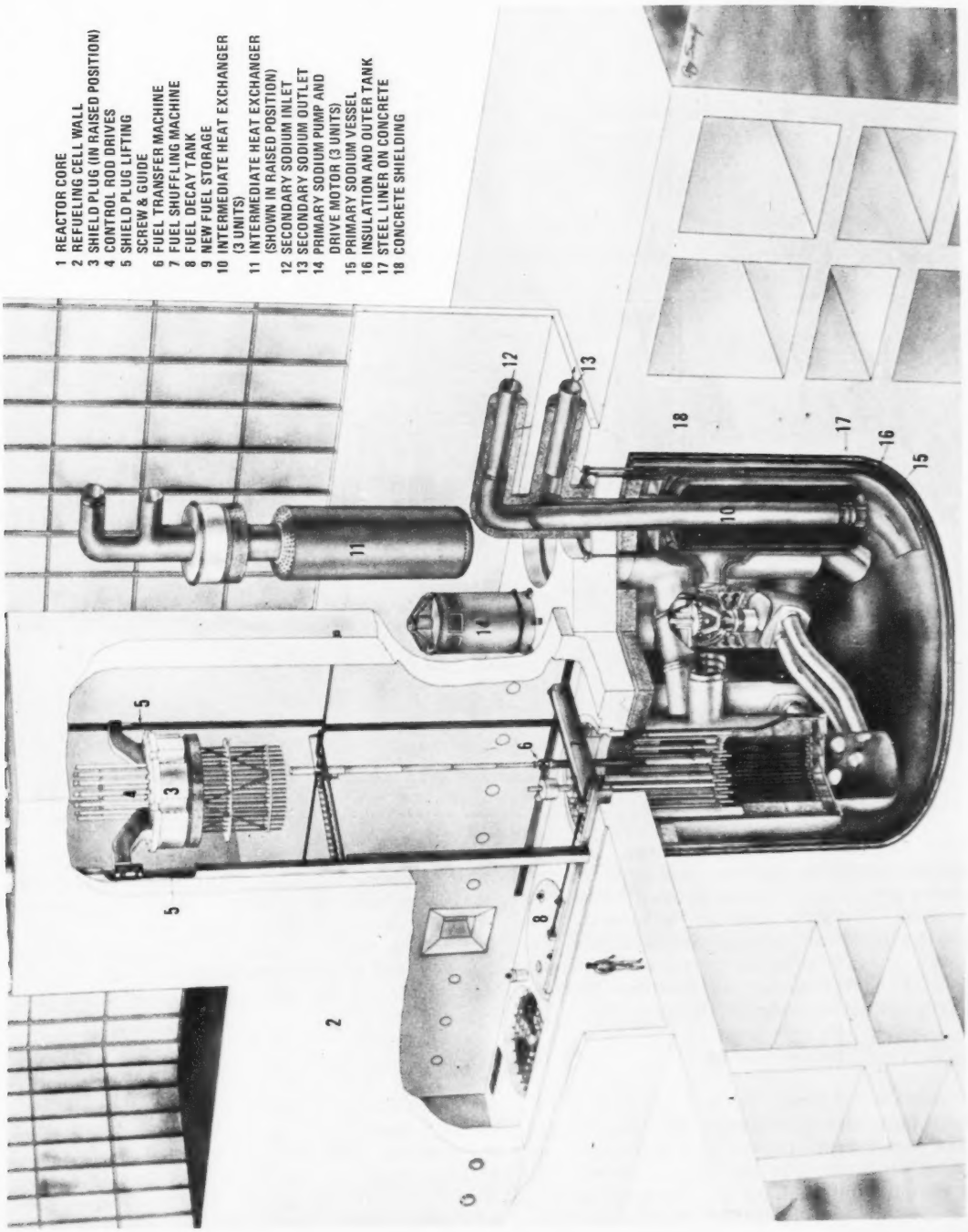
1 REACTOR CORE
2 REFUELING CELL WALL
3 SHIELD PLUG (IN RAISED POSITION)
4 CONTROL ROD DRIVES
5 SHIELD PLUG LIFTING SCREW & GUIDE
6 FUEL TRANSFER MACHINE
7 FUEL SHUFFLING MACHINE
8 FUEL DECAY TANK
9 NEW FUEL STORAGE
10 INTERMEDIATE HEAT EXCHANGER (3 UNITS)
11 INTERMEDIATE HEAT EXCHANGER (SHOWN IN RAISED POSITION)
12 SECONDARY SODIUM INLET
13 SECONDARY SODIUM OUTLET
14 PRIMARY SODIUM PUMP AND DRIVE MOTOR (3 UNITS)
15 PRIMARY SODIUM VESSEL
16 INSULATION AND OUTER TANK
17 STEEL LINER ON CONCRETE
18 CONCRETE SHIELDING

Fig. 9. 1000-MW(e) LMFBR with refueling cell (GE photo).

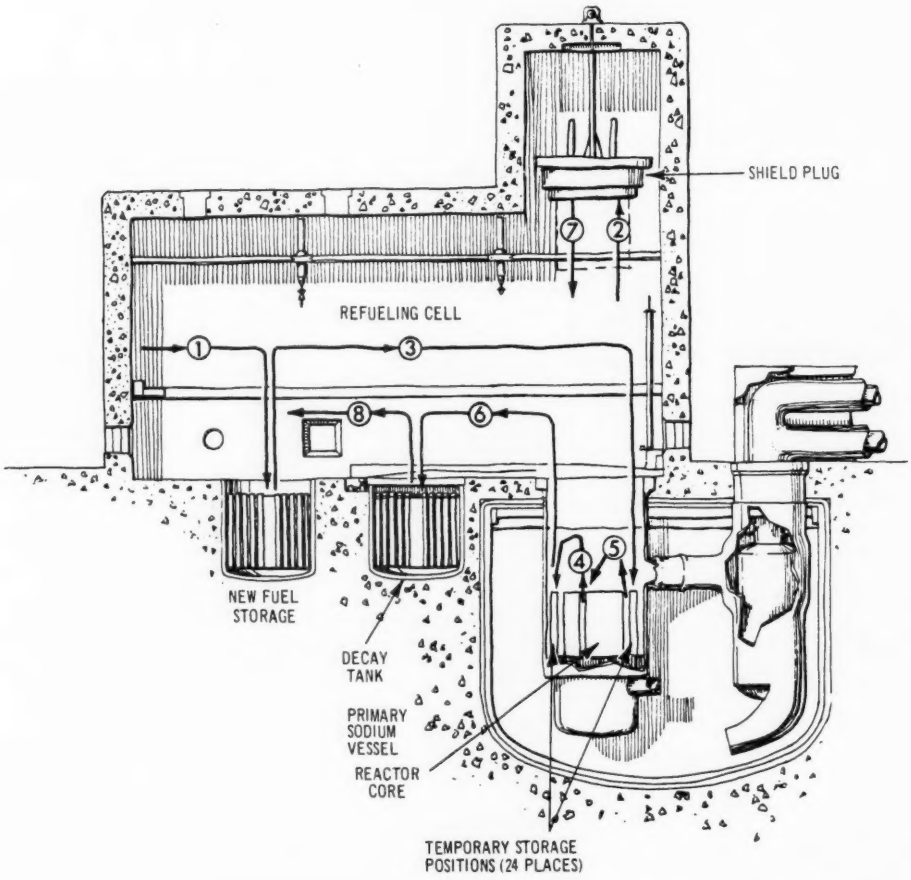

Fig. 10. Refueling schematic diagram (GE photo).

50,000 to 100,000 fuel elements in a reactor, it is quite probable that fuel failures will occur rather frequently and release fission product activity to the system. It may be impractical to avoid this even with a rapid fuel handling system. Therefore, there is a direct interaction between fuel handling, contamination of the primary system, shutdown cooling, plant maintenance, and plant factor.

MAINTENANCE

Maintenance of power reactor systems is extremely important to achieving high plant factor and low power cost. Maintenance philosophy and requirements for sodium- and helium-cooled systems are quite different and may be quite important in the ultimate competitive position of these plants.

Because of the activation of sodium, the entire primary sodium system and all components in it must be heavily shielded. Also, since the half-life of ^{24}Na is reasonably long (~15 h), maintenance must be performed on a highly radioactive system. As a result, components must be designed for removal and decontamination before significant work can be done on the parts of the unit exposed to sodium.

By contrast, helium is inert and maintenance can be performed directly on system components. The GCFR design developed by GGA (Fig. 4) is provided with access ports to permit entry into the end cavities containing the primary circuit components. The reactor cavity is adequately shielded from the component cavities and access is possible after the system has been depressurized and cooled. To accomplish maintenance in this manner, however, will require that the components and associated systems not become con-

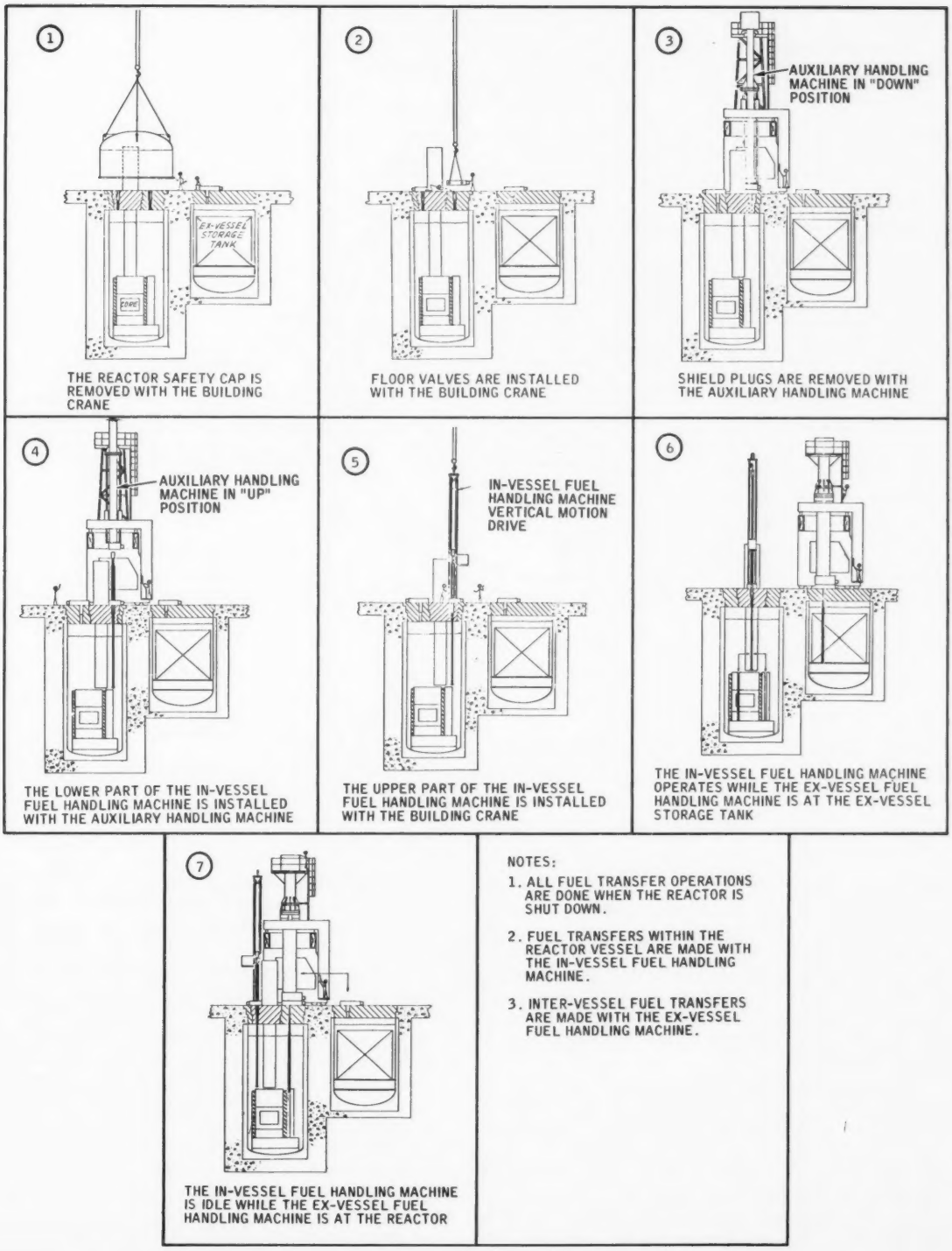
1
EX-VESSEL STORAGE TANK
CORE
THE REACTOR SAFETY CAP IS REMOVED WITH THE BUILDING CRANE
2
FLOOR VALVES ARE INSTALLED WITH THE BUILDING CRANE
3
AUXILIARY HANDLING MACHINE IN "DOWN" POSITION
SHIELD PLUGS ARE REMOVED WITH THE AUXILIARY HANDLING MACHINE
4
AUXILIARY HANDLING MACHINE IN "UP" POSITION
THE LOWER PART OF THE IN-VESSEL FUEL HANDLING MACHINE IS INSTALLED WITH THE AUXILIARY HANDLING MACHINE
5
IN-VESSEL FUEL HANDLING MACHINE VERTICAL MOTION DRIVE
THE UPPER PART OF THE IN-VESSEL FUEL HANDLING MACHINE IS INSTALLED WITH THE BUILDING CRANE
6
THE IN-VESSEL FUEL HANDLING MACHINE OPERATES WHILE THE EX-VESSEL FUEL HANDLING MACHINE IS AT THE EX-VESSEL STORAGE TANK
7
THE IN-VESSEL FUEL HANDLING MACHINE IS IDLE WHILE THE EX-VESSEL FUEL HANDLING MACHINE IS AT THE REACTOR
NOTES:
1. ALL FUEL TRANSFER OPERATIONS ARE DONE WHEN THE REACTOR IS SHUT DOWN.
2. FUEL TRANSFERS WITHIN THE REACTOR VESSEL ARE MADE WITH THE IN-VESSEL FUEL HANDLING MACHINE.
3. INTER-VESSEL FUEL TRANSFERS ARE MADE WITH THE EX-VESSEL FUEL HANDLING MACHINE.

Fig. 11. In-plant fuel handling sequence (AI photo).

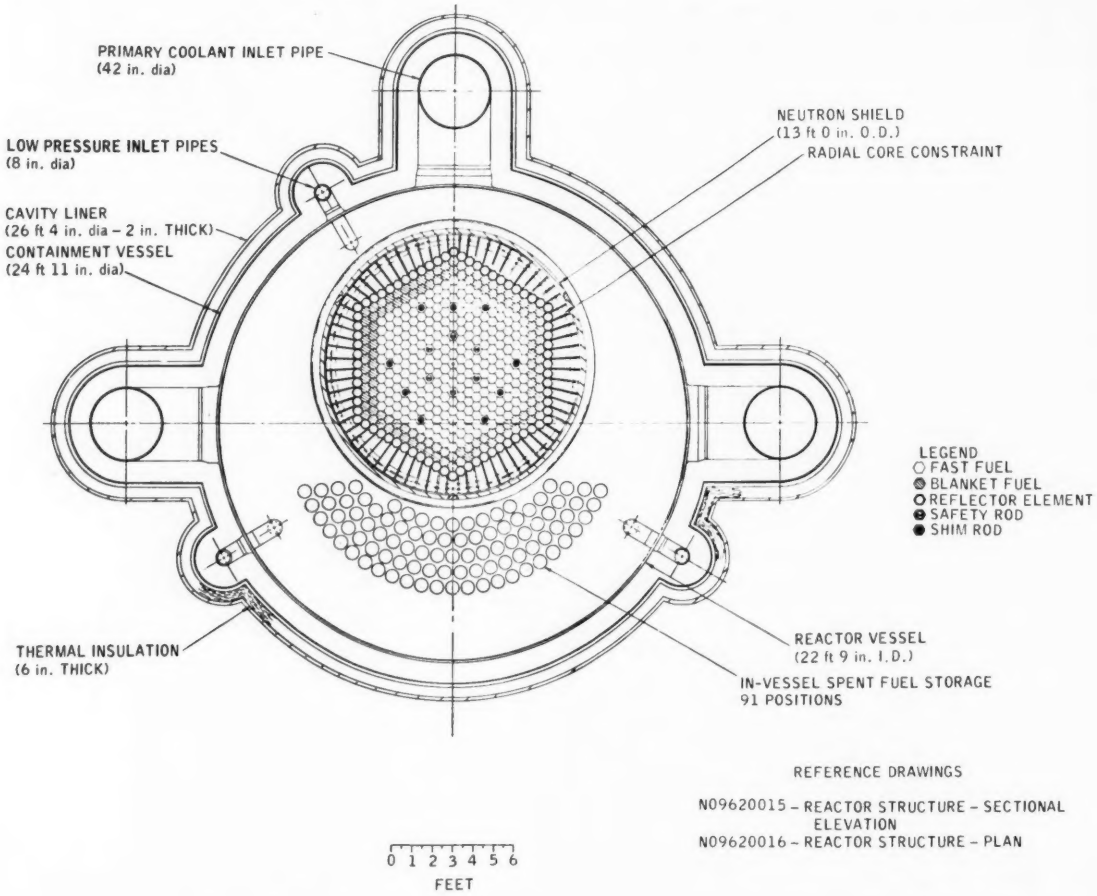

Fig. 12. Reactor structure—cross section 2-plug—eccentric core 1000 MW(e) (AI photo).

taminated. This will impose rather stringent requirements on system contamination, and probably preclude consideration of fuel elements vented directly to the coolant. Figure 16 is a photograph of a model of the boiler arrangement in the GCFR (Fig. 4). Maintenance will obviously be quite difficult even with direct contact and may be prohibitively difficult if the system becomes highly radioactive.

Increased attention is being given to designing systems and components for ease of maintenance. A variety of maintenance and repair operations have been required on LMFBR systems to date. Several large components have been removed, cleaned, and repaired by procedures similar to those employed on the EBR-II primary sodium pumps (Fig. 17). Similarly, the GE design (Fig. 9) includes extensive provisions for the removal, cleaning, and repair of the primary sodium pumps and the intermediate heat exchangers. A heat exchanger is shown in the raised position, but the container has been omitted from the figure for clarity. Access to and removal of the pumps and heat exchangers were major considerations in this design concept which places all these units outside the refueling cell and simplifies maintenance.

A more recent GCFR concept developed by GGA would permit component removal for maintenance. Since helium does not present a chemical activity and cleaning problem, component removal and cleaning should be easier. On the other hand, the steam generator may be a difficult component to move, handle, decontaminate, and repair because of the heavy structure required for high-pressure operation.

A very important factor in assessing maintenance of fast power reactors involves the levels of radioactive contamination that can be tolerated.

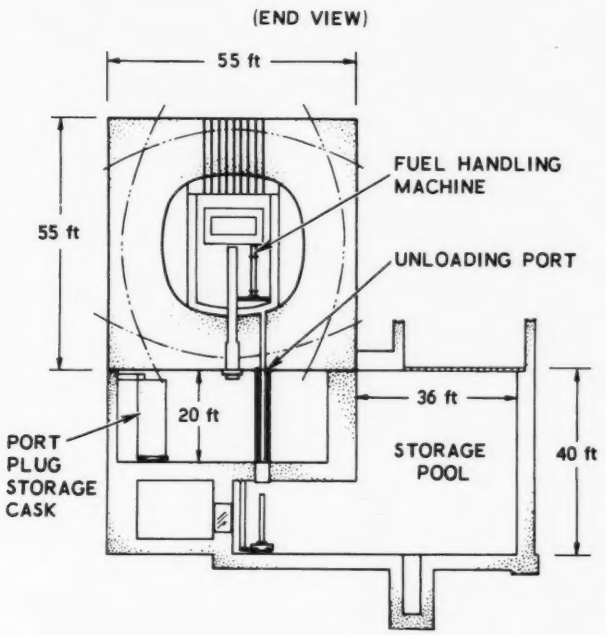

Fig. 13. Fuel handling facility (GGA photo).

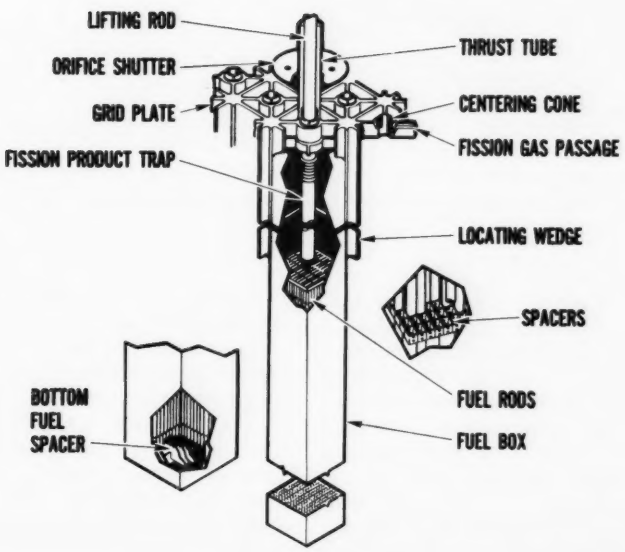

Fig. 14. Fuel element (GGA photo).

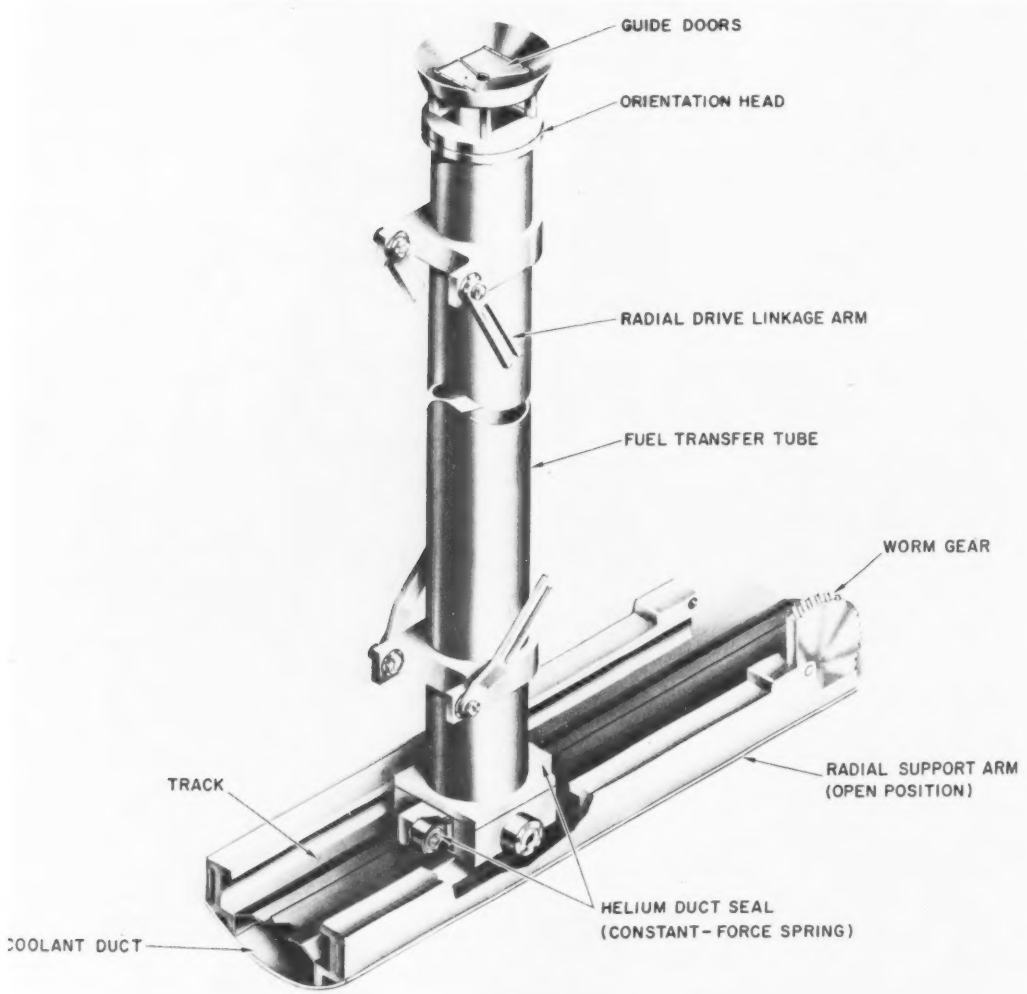

Fig. 15. Fuel transfer tube and portion of bridge (GGA photo).

There may be a very strong incentive to vent fuel elements. This may increase the permissible burnup by reducing the pressure stress in the fuel element and may eliminate a potential failure propagation mechanism resulting from rupture-type failure of fuel elements. If these potential advantages of vented fuel do in fact materialize, they should benefit both the LMFBR and the GCFR; however, exploitation of vented fuels may be limited by maintenance considerations. Therefore, just as the system design may be controlled by shutdown cooling considerations, it may also be controlled by maintenance considerations.

COMPONENTS

Although the operating requirements for LMFBR and GCFR components are very demanding, they appear to be amenable to engineering development and are within the accepted limitations of engineering extrapolation. For the LMFBR, the sodium-to-water/steam generator appears to be the most difficult component. EBR-I, EBR-II, the Dounreay Fast Reactor, SRE, and the Hallam reactor, all employed double-tube steam generators. Although these units represent a variety of concepts and designs, they all have

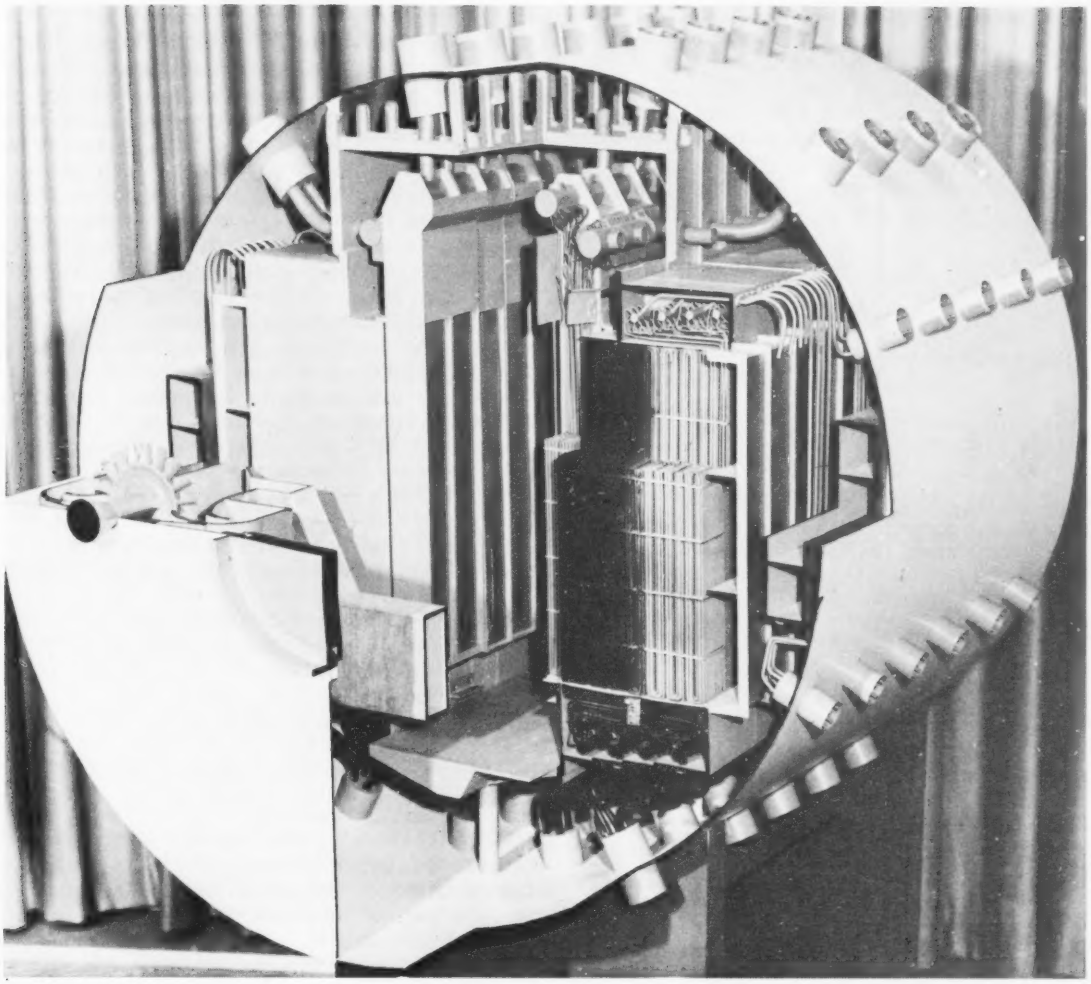

Fig. 16. Model of GCFR boiler (GGA photo, courtesy of Sulzer Bros., Ltd., Winterthur, Switzerland).

operated reasonably well. On the other hand, steam-generator and reactor manufacturers have concluded that double-tube units would be prohibitively expensive for commercial LMFBR's. As a result, there is strong interest in developing reliable, inexpensive, single-tube steam-generator designs. The Fermi reactor utilizes a single-tube design, but these units have experienced considerable difficulty with leaks. Although the causes of these difficulties have been thoroughly evaluated and there is a strong belief they could be avoided in the future, there is as yet no successful experience with single-tube steam generators in sodium reactor systems. It should be noted that the UKAEA is using single-tube steam-generator units on their Prototype Fast Reactor now under construction.

The GCFR does not require components with basic incompatibilities such as presented by the sodium-to-water system. All the components required for the GCFR should be amenable to engineering extrapolation. There may be problems related to the design of these components to meet installation and maintenance requirements in the confined and restricted spaces that will be available within the PCRV. It is quite likely that the

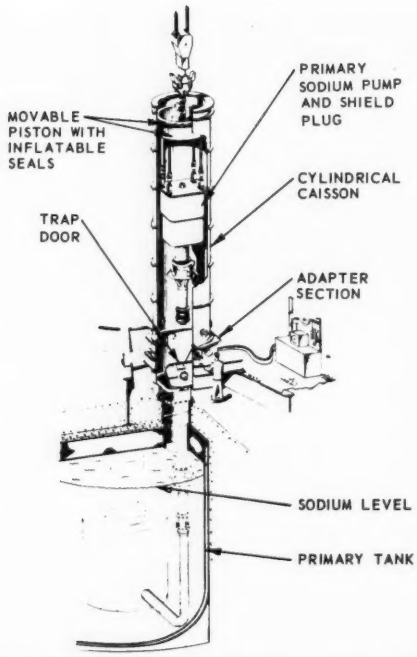

Fig. 17. EBR-II primary pump removal (ANL photo).

steam generator also may be the most difficult component for the GCFR, but for quite different reasons than for the LMFBR. Although the moisture addition to the helium system that would result from a small steam leak can be tolerated, it will be necessary to terminate operation and drain the unit rather quickly. The leakage rate through a break in a high-pressure system increases rather quickly and the introduction of water to the primary system is deleterious. If the unit can be completely isolated it should be possible to operate at reduced power until convenient to repair, but it would seem that it will be desirable to repair the unit as quickly as possible. Maintenance of such a unit in a confined space may be quite difficult (Fig. 16). These high-pressure units will be rated for ~2500 psi on the steam side, and 1500 psi on the helium side, and will operate at temperatures in excess of 1000°F. Designing these units to permit access for tube plugging and other repair within the PCRV will be extremely difficult. Designing them for removal and repair will also be difficult. If they become contaminated with fission products to the extent that remote or semi-remote maintenance is required, the job will become even more difficult. It would appear that the steam generators for the GCFR should be designed to the same high standards of reliability as required for the LMFBR.

CONCLUSIONS

Fast breeder reactors are potentially low-power-cost energy sources because of their unique capability to breed, which makes possible very low fuel costs. They are technically feasible and the predicted performance of the various fast reactor systems should be attainable. It is necessary, however, that this performance be on a reliable and predictable basis during the entire service life of the plant. This involves plant factor and availability factor during the 30-plus-year life of the plant. The performance, reliability, maintainability, and replaceability of plant systems and components will, in large measure, determine these performance factors. (While it is true that other operating variables, such as allowable burnup in the fuel, can also affect these operating factors, the effect is reasonably predictable and is comparatively small.)

Selection of a fast breeder system must include an objective and realistic evaluation of the operability and reliability of the systems and components under all conditions that are anticipated during the service life of the plant. An assessment of the LMFBR and GCFR systems and components in this context, indicates that the LMFBR is inherently more capable of accommodating probable off-normal conditions. This results largely from the basic requirements and system characteristics for normal conditions. A brief comparison of the two systems identifies the differences between the systems and their impact.

1. Both reactor types operate at high power density and cannot tolerate loss of coolant. To ensure this, the pressurized GCFR places all of the primary helium system within the pressure envelope (the PCRV). Because helium is compatible with steam and water, no intermediate heat transfer system is needed and the boiler becomes a component in the primary system. The consequences of a leak, and the requirements for maintenance or replacement, must be evaluated within the environment of the primary coolant system. The LMFBR pot design reflects a similar system philosophy, but present a quite different situation because of the separation of problems which result from the secondary sodium system. (An LMFBR without a secondary system would present much more difficult problems than the GCFR.) The steam generator and all steam system components are completely divorced from the radioactive primary system.

2. Even though helium is compatible with water and steam, and sodium is not, the GCFR steam generator must be as reliable as the LMFBR steam generator. Also, it is quite probable that maintenance of GCFR steam generators will be

more difficult because they will be contaminated on the primary coolant side.

3. The primary helium coolant system is basically a "clean system" because it does not activate and because it is chemically inert. The GCFR concept exploits this characteristic by utilizing maintenance and operating procedures appropriate to clean conditions. This, of course, leads to simplification of the GCFR concept but also requires that a clean system be maintained. The primary sodium coolant system is not a clean system in this context since it activates and is chemically very active. The LMFBR concept must accommodate these characteristics. Since many off-normal conditions can cause radioactive contamination of the primary system, the LMFBR is inherently more capable than the GCFR in accommodating such conditions.

4. Fission product decay-heat removal is a very special problem in high power density reactors. Because this source of heat will continue under all circumstances, its removal must be assured under all circumstances. Redundancy of systems, power sources, and controls does not necessarily satisfy the requirements in the same sense that this can be done for scram systems or other systems that must perform a function very reliably. (Redundant fail-safe systems usually assure that something not be done—like a circuit will not remain closed, if this is not desired.) Shutdown cooling of the GCFR requires the continued forced circulation of coolant to avoid fuel damage. This situation leaves the GCFR vulnerable to the "but what if" type of questions regarding reliability. Fuel submerged in liquid sodium represents a far better situation than fuel submerged in helium gas. It provides a shutdown cooling environment which is inherently far superior. These characteristics make the LMFBR better able to accommodate off-normal shutdown cooling conditions.

The design of systems and components for fast reactors involves a very complex and sophisticated technology. The requirements for their reliable and efficient operation are very demanding. Their capability to accommodate or tolerate off-normal conditions may be controlling and must be carefully evaluated in the process of plant selection.

ACKNOWLEDGMENTS

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FUEL CONSIDERATIONS

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INTRODUCTION

It has been already noticed that the fuel characteristics of the fast reactor prototypes now under construction or being constructed in the 1970's are remarkably similar. The fuel material is the same (mixed oxide), the canning material is stainless steel (316 or 318 Nb), and the principal operating characteristics are very close (linear power 400 to 450 W/cm; minimum burnup value 40,000 to 50,000 MWd/tonne).

Since these prototypes are considered a technical rather than an economic demonstration, fuel characteristics have been selected in favor of safe and reliable operation and do not represent the estimated attainable limits.

The next step is represented by the large power plants to be built in the next decade; the fuel characteristics are, at present, mainly determined by economic considerations: burnup values of the order of 100,000 MWd/tonne and a heat rating 500 W/cm for mixed oxide are considered necessary. The choice of fuel and canning material is also thought to broaden, leading to better chances to attain the economic targets.

I shall limit myself to some considerations on important fuel parameters: burnup and heat rating.

BURNUP

Large irradiation programs have been undertaken in fast and thermal reactors. At present the situation may be summarized as follows:

1. Sodium-bonded metal fuels (U-Pu-Zr alloys) have been irradiated in EBR-II to 6.7% burnup. Various canning materials were used: 304 SS, 316 SS, Hastelloy, and vanadium alloys. Capsule irradiations have been performed on the same fuel material up to 12.5% burnup.

2. For oxide fuels (UO_2-PuO_2), two important sets of results have been obtained in DFR and in EBR-II. Vibrocompacted and pelletized fuels were irradiated at burnups of 7 to 8% heavy atoms, without can failure. In capsule irradiations, small prototype fuel rods sustained significantly higher burnups in thermal fluxes.

Test fuel subassemblies in Rapsodie attained 4.8% burnup, and the driver fuel itself has now reached 4.5% in the central part of the core without can rupture.

3. A number of vibratorily compacted carbide fuel pins were irradiated up to 7 at.% in EBR-II; high density pellets attained 10 to 11 at.% in thermal flux irradiations.

In the high burnup race, the difficult point is known to be the canning resistance. Among the three main stresses applied to the can—thermal, fission gas pressure, and fuel swelling stresses—the latter appears to be the most important for the fuel behavior. Fission gas pressure and thermal stresses can be calculated with a reasonable accuracy and thus taken into account in the design of a fuel rod. No theory is available at present for the evaluation of canning stresses due to swelling in the fuel material.

The fission gas pressure depends on the quantity of fission gas released from the fuel. A number of data are now available which allow reasonable prediction.

In metal fuels,[1] experimental data indicate that fission gas release is largely independent of burnup and temperature, and depends principally on the amount of fuel swelling. When the necessary voidage is provided to accommodate the fuel swelling, about 60% fission gas release is observed.

In oxide fuels, the fission gas release depends on both the heat rating and burnup. Divided forms of oxide, like vibrocompacted powders, exhibit

greater fission gas release than sintered pellets. For heat ratings of 400 to 500 W/cm, and burnups larger than 50,000 to 60,000 MWd/tonne, high values like 80% are obtained, and for practical design of fuel elements it may be assumed that 100% of the fission gases are released during the entire life. Overestimating the fission gas pressure at the beginning of irradiation is unimportant since the creep of the can is negligible at this period.

High fission gas release may be considered beneficial in the case of gas- or steam-cooled reactors since it provides an internal pressure that counteracts the external pressure of the coolant. In several fuel pin designs, this internal pressure is increased by pressurization of the cans to avoid creep buckling of the plenum and possible creep of the oxide itself.

Solutions proposed to minimize the creep of the can under fission gas pressure are mainly to reduce this pressure by locating the plenum at the cold end of the fuel rod, i.e., at the coolant inlet, and to improve the creep characteristics of the canning material. This can be done in stainless steels by minor alloying additions or by various mechanical and heat treatments. In some cases, and especially for gas-cooled reactors, better creep characteristics are required and nickel alloys are considered desirable.

Suppression of fission gas pressure can be expected from utilization of vented fuels. This elegant solution, however, does not seem to be adapted to the near future and the probable reason is that fission gas pressure is not the limiting factor of the life of a fuel rod, compared to fuel swelling. Furthermore, apart from the suppression of pressure stresses in the cladding, the main advantage brought by vented fuels arises from the possibility of shortening the fuel pins, which would lead to a reduction in the construction cost of the reactor. This reduction has to be balanced against cost increments due to the presence of fission products in the coolant. Finally, in most cases, because of compatibility problems between the fuel material and the coolant, the venting device must be perfectly reliable. Several solutions have been studied but their development needs further experimental support.

As far as swelling is concerned, most of the experiments performed to date were directed to determination of "inexorable" swelling rate and swelling in fuel elements as revealed by pin diameter and fuel column length increases.

In metal fuels, it has been shown[1] that fuel swelling can be contained by stainless steel or Hastelloy canning of reasonable thickness, provided a certain amount of initial free swelling is allowed (30 to 40%). In this case, a substantial quantity of fission gases escape from the fuel, thus reducing the swelling due to gaseous fission products. The plasticity of alloys at a temperature near 700°C seemingly favors deformation of the fuel material with relatively low stresses in the cladding.

In oxide fuels, the swelling is taken into account by leaving a certain amount of voids distributed in internal porosity and fuel-to-clad gap. In this model, the value of the smeared density necessary to obtain a given burnup is dependent on the swelling rate and on the fraction of the initial porosity that cannot be filled by the fuel material without excessive stresses in the cladding.

Estimations of the swelling rates due to gaseous and solid fission products have been made and can be considered reasonably accurate.[2]

On the other hand, irradiation of a large number of prototype fuel pins with various smeared densities and heat ratings have been performed in thermal and fast reactors, and striking differences appeared between the two sets of experiments. In thermal-neutron fluxes, high burnup irradiations (~10 at.%) did not reveal any significant fuel diameter increase.[3]

In fast-neutron flux experiments, fuel-rod diameter increases were noticed at burnup in the range 4 to 5 at.%, can failure occurring at ~7 at.%. The rate of diameter increase (~0.2%/percent burnup) is consistent with swelling rates observed, but no clear relation between the initial voidage and the beginning of diameter increase is apparent.

Interpretation of these results is not easy since the observed variations in diameter result from fuel swelling itself, voidage decrease, and can swelling. It seems, however, that the situation is getting clearer since preliminary results became available on the swelling of stainless steels, and that the contribution of this swelling to the total deformation may be very important in a fast-neutron environment.

In carbide fuels where fission gas release is lower than in oxides, swelling rates are greater by a factor of 4 to 6, and to accommodate this greater swelling, initial voidage in the fuel must probably be more important than in oxide fuels. In this respect, sodium bonding is an advantage since it may provide a significant space for swelling without high fuel temperatures or clad restraint.

The canning material, however, remains the most important part of the fuel element. Creep resistance, creep ductility under irradiation, and swelling are the main parameters to be investigated. Up to now, major attention has been given to stainless steels because of their commercial availability and good corrosion properties. Other alloys like Incoloy, Hastelloy, or vanadium alloys

are considered, essentially for their higher mechanical characteristics.

Experimental data show that the creep rate of stainless steels is not altered after irradiation in a fast flux. On the other hand, since creep is a vacancy mechanism, it may be influenced by the magnitude of the neutron flux.

Loss of ductility under high fast neutron doses remains the most important factor since it is a determinant for can failure, the relative weakness of available fast-neutron fluxes is a real problem in this field, and very little data have been obtained at fluences in the vicinity of 5×10^{22} n/cm^2. Cladding failures in experimental fuel pins exhibited elongations of 1% or less and, here again, it must be noticed that most irradiation experiments that are feasible in actual fast reactors are not truly representative, since, even though high burnups can be attained in the fuel material, the neutron fluences on the cladding are significantly lower than expected in large fast reactors.

Finally, the time required to perform extensive work on this field is such that the fast reactor prototypes will probably be in operation before a sufficient amount of data is obtained to ascertain our knowledge, which emphasizes the technical importance of these prototypes.

Cavity formation and associated swelling have been discussed extensively during this meeting. Relative volume changes in 304 stainless steel as high as 1% for fluences of 4×10^{22} n/cm^2 were reported, and rough extrapolation of the results indicates that volume changes in the vicinity of 10% could be observed for fluences near 4×10^{23} n/cm^2.

It is not necessary to underline the importance of this phenomenon and the need for further studies of the magnitude of swelling as a function of neutron flux and fluence, the influence of cavities on mechanical properties, and the sensitivity of various alloys to swelling.

Stainless steel is the cladding material most commonly adopted in projects, the maximum can temperature being 650 to 700°C, as calculated by hot channel factors. In this temperature range, mechanical characteristics of stainless steels drop rapidly and even a reduction of 20 to 30°C of the maximum can temperature would be appreciable, allowing a reduction of can thickness or expansion space. A better knowledge of the different factors involved, especially those related to mixing of the coolant in the subassembly, would probably be beneficial in this field.

LINEAR POWER

Limiting values of the linear power have been investigated for different fuel materials.

Metal fuels[1] possess good thermal conductivity but relatively low melting points. Successful irradiations have been performed at linear power of 340 W/cm in EBR-II; experiments are in progress or in preparation at 500 to 650 W/cm, with can temperatures 600 to 630°C.

Oxide fuels[2] have been tested at linear power as high as 580 W/cm, the limiting value corresponding to central melting being about 600 W/cm. In the case of oxides, central melting is not discarded at least for short periods in the life of the fuel element, and the upper limit could be somewhat higher.

In carbide fuels, the limiting factor is not the melting temperature, but the swelling. Irradiation of carbide fuel rods in EBR-II at linear power of 920 W/cm and 3.6 at.% burnup has been reported.[4]

Provided a sufficient amount of internal void is left to accommodate the swelling, evaluation of carbide performance shows that with helium-bonded carbide fuel rods, linear power in the range 600 to 1000 W/cm can be attained. For sodium-bonded, because the fuel temperature is decreased, an increase of linear power to 1500 W/cm is possible. In this case, the limiting factor is set by the thermal stresses in the can and the possibility of transferring large quantities of heat to the coolant.[5]

Compared to the figures quoted above for oxides, linear powers adopted for reactors in construction are modest (400 to 450 W/cm for oxide fuels).

This comparison reflects the uncertainties existing about operating parameters that lead to large hot-channel factors.

Among these parameters, some are relevant to the reactor itself (reactor power, flux distribution); the others are related to the fuel (heat conductivity of the fuel material, heat transfer coefficient between can and pellet).

Uncertainties on reactor power and flux distribution in the core appear to be important; in the case of oxides, they correspond to a reduction of about 100 W/cm of the maximum admissible linear power. It is hoped that present evaluations are pessimistic and that it will be possible to reduce them for large reactors, thus providing a gain of about 50 W/cm.

The thermal conductivity of oxides has been studied extensively by in-pile and out-of-pile experiments. Its variation with temperature, stoichiometry, and porosity is known with a reasonable degree of accuracy. The heat transfer coefficient between cladding and fuel is dependent on the diametral gap and gaseous atmosphere in the can. With helium filling, initial values of 0.6 to 1.6 W/(cm^2°C), corresponding, respectively, to gap values of 0.2 to 0.06 mm, have been founded.

Examination of Rapsodie fuel pins revealed heat transfer coefficients in the range of 0.6 to 0.7 W/(cm^2°C) for an initial gap of 0.2 mm. With argon atmospheres, somewhat lower values are reported [0.4 W/(cm^2°C) for a 0.2-mm gap].[6]

Variations of this coefficient with burnup are difficult to evaluate. They result from variations of the gap and pollution of the gas by fission products. For heat ratings of 400 to 500 W/cm, it seems that the final value is rather insensitive to the initial gap and lies in the range of 0.6 to 0.8 W/(cm^2°C). The uncertainties on the heat transfer coefficient have an important effect on the fuel element design, and a rough estimation shows that a reduction of these uncertainties could afford a gain of 50 to 100 W/cm on the admissible linear power.

CONCLUSIONS

Present evaluation of oxide fuel performance shows that linear powers in the range of 500 to 550 W/cm can be envisaged for large power plants.

For metal alloys, the admissible heat rating seems to be the same order of magnitude, although full demonstration has not yet been achieved.

For carbides, it is estimated that linear powers of 700 to 1500 W/cm will be attainable, the latter with a sodium bond, which is presently considered as an advanced concept still in an early stage of development.

To attain the high burnups required for fast reactors, it appears that important problems remain to be solved.

Concerning the fuel material, the minimum amount of porosity necessary to accommodate the swelling, the optimum void distribution and the stresses exerted on the cladding need to be determined.

For the cladding, knowledge of the creep ductility at high neutron fluences, and of the swelling and its influence on mechanical properties will be determinant for the fuel element design.

For some of these problems and especially those related to the cladding, uncertainties will remain until operation of the prototypes starts. This underlines their importance for the development of fuel elements.

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FAST REACTOR CORE HEAT REMOVAL

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INTRODUCTION

The question of heat removal is identical to the question of the best method for fast reactor *cooling*, but different from the question for the best fast reactor *coolant*. The latter question is much broader and includes not only thermohydraulic qualities, but physics characteristics and the implications of the whole primary circuit. This paper concentrates mainly on problems of the first kind, namely, core cooling, but occasionally we must also consider the general coolant discussion. I shall try to explain the thermal problems without cutting off other considerations.

Our problem is stated as follows: Given a parallel array of metal-clad rods filled with PuO_2-UO_2 of 80 to 85% TD, with an axial blanket part at either end and a fission gas plenum at one end, and given maximum permitted temperatures for fuel and cladding: *Arrange and cool this in a way that none of the maximum temperatures is exceeded and that the power generating costs will have a minimum.* Naturally a number of boundary conditions have to be fulfilled, of which I mention only safety and operational reliability. I shall touch on them occasionally.

The dependence of the power generating costs on core design is shown schematically in Fig. 1. The normal expressions for power cost primarily depend on the parameters listed in the second column. We neglect the dependence on nontechnical parameters such as interest rates, load factors, plutonium price, unit costs for fabrication, transport, reprocessing, etc. The primary parameters themselves are dependent on a number of core design parameters, as listed in column 3. We cannot explore the details of these interdependences in this paper, but we shall draw some lines of practical understanding. We will call these core parameters "internal" parameters.

	Among Others, Function of	Among Others, Function of	
Power Generating Costs	Burnup Bu	Inlet temperature	ϕ_1
		Temperature rise	$\Delta\phi$
		Core height	h_c
		Core diameter	d_c
	Rating γ	Total channel length	h_{tot}
		Plenum length	h_p
		Rod diameter	d_r
	Breeding ratio BR and other fuel cost parameters	Clad wall thickness	s
		Coolant channel area	F_c
		Pressure drop	Δp
		Fuel density	ρF
		Hot channel factors	H
	Efficiency η	Maximum clad temp	T_{max}
		Rod power	$q\gamma$
		Enrichment	e
	Plant costs C		

Fig. 1. Cost dependence on "internal" or core parameters.

Also, to an even larger extent, the power costs depend on the design and components of the coolant circuit and the total plant outlay as shown in Fig. 2. We will define these parameters as "external" parameters.

Mainly, three coolants are being discussed for fast reactors: sodium, steam, and helium (or CO_2). Sodium has the best cooling potential by far, and turns out to be very favorable with respect to the internal parameters. On the other hand, it has not yet been demonstrated that the external parameters, especially the steam generator, will favor a low cost solution.

Steam is just the opposite: The internal parameters are unfavorable, resulting in low breeding, low burnup, moderate efficiencies, and many problems with corrosion, safety, tolerances, etc. On the other side, it is expected that the external parameters, based on light-water reactor tech-

	Among Others, Functions of	Among Others, Functions of
Power Generating Costs	Plant costs C	Heat exchanger Steam generator Turbine Generator Pumps
	Efficiency η	Valves Safety Building

Fig. 2. Cost dependence on "external" or circuit parameters.

nology, will allow for low cost solutions. This, too, has not been demonstrated and it might take a long time, since in the USA the steam activities have been reduced and they are under reconsideration in Germany.

Gas offers some advantages and disadvantages for both the internal and external parameters. It probably has very good future potential in the long range and may profit from the HTGR experience. At present, all gas work is at a very early stage and looks prosperous, but many details have to be cleared before it can be evaluated as sodium is today.

We will discuss all three coolants in the following sections. Before doing so, we already can fix two parameters of the third column of Fig. 1: the rod power q_r and the rod diameter d_r.

For fast reactors, the relation between q_r and the conductivity integral

$$q_r = 4\pi \int_{T_1}^{T_0} k\, dT (\mathrm{W/cm}) \quad , \qquad (1)$$

where

T_0 = fuel center temperature

T_1 = fuel surface temperature

k = thermal conductivity of fuel

is quite exact. The economic optimum for sodium cooling is the maximum rod power, where T_0 comes close to the fuel melting temperature.[5] It has been shown that this is also true for steam cooling,[2,3] so that (neglecting the weak dependence on T_1) $q_r \approx 500$ W/cm at the hot spot; the average value of q_r is then given by consideration of hot channel factors and power distribution. Carbide fuel has been proposed for sodium and gas cooling and offers some advantages, but rod powers[5] of 1000 to 1500 W/cm are required for economic and technical reasons.

The rod diameter d_r in all designs turns out to be 5 to 6 mm as a result of fuel-cycle cost optimization between fabrication costs and plutonium interest rates, quite legally neglecting other dependences.[5,6] Since this is about half the value of PWR's, the heat flux has at least double the value, amounting to 200 to 300 W/cm^2. For carbide fuel rods, larger rod diameters are more economic[4] and technically feasible because of the large heat fluxes at the surface.

SODIUM

Temperature Limits

The maximum fuel temperature in connection with the rod power was considered in the preceding paragraph. The thermal conductivity k depends on fuel density, and this is fixed by swelling with burnup. Karsten[7] has combined the existing knowledge into a simple swelling model where he assumes a "plastic" fuel zone ($T > 1700$°C), a "creep" zone ($T = 1300$ to 1700°C), and a "cold" zone ($T < 1300$°C). To avoid an extensive swelling pressure on the cladding, the fuel density for burnups up to 80,000 to 100,000 MWd/ton has to be in the range of 80 to 82% of the theoretical value. (Compared to earlier designs with 90% TD, this has resulted in a reduction of breeding ratios by 0.05 with any coolant.)

The maximum clad temperature is determined by mechanical criteria. The maximum material stress occurs at the end of the fuel lifetime and is given by the combination of fission gas pressure, fuel swelling pressure, and thermal stress (Fig. 3). This may lead to pin failure by one of the following mechanisms:

1. Exceeding the yield strength in the 650 to 750°C temperature range will result in clad failure.[9]
2. Strain cycling may result in fatigue failure.

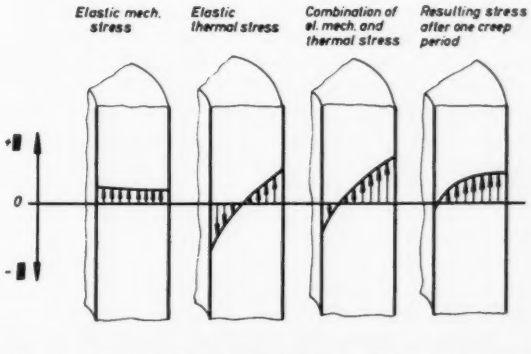

Fig. 3. Combination of mechanical and thermal stress in fast reactor claddings.

3. For irradiated material, the rupture strain is in the range of 1% (high-temperature embrittlement). Therefore, creep deformation up to this value must be avoided.[8] For most designs this determines the maximum allowable cladding temperature.

But there is still another mechanism:

4. The combination of creep and strain cycling results in a certain type of thermal ratcheting. This again leads to failure above a certain strain, limited by irradiation effects. Compared to the continuous creep mentioned under (3), this effect is the most dangerous. While thermal stresses are degraded in the continuous case, they reappear partially in the cyclic case. Therefore this effect normally defines the limiting value for the fuel element design and the limiting temperatures.[10]

Thus, in general, by one or the other of these criteria, a certain maximum fission gas and fuel swelling pressure is given. Since the fission gas pressure also depends on the length of the fission gas plenum, there is no unique relation, for example, between burnup and clad temperature. Rather, the allowable can temperature is a function of

1. Burnup (amount of fission gas release and fuel swelling)
2. Clad material
3. Wall thickness (as low as possible for physics reasons)
4. Length of plenum h_p
5. Number of power cycles during fuel lifetime
6. Heat flux in cladding.

Figure 4 shows the permissible fission gas pressure for the cladding X8-Cr-Ni-Mo-V-Nb-1613 as a function of wall thickness at 700°C as limited by the thermal ratcheting criteria according to a theory developed by Schmidt[11] [criterion (4)] and according to criteria (1) and (3).

The following assumptions have been made: T_{clad} = 696°C; number of cycles = 144; swelling pressure according to Karsten[7] ρ_f = 80% TD.

However, the creep rate under irradiation is not well known. Therefore, the dotted line shows the allowable pressure under the assumption of a creep velocity 10 times larger than the unirradiated value. For the unirradiated material, this corresponds to a 35°C rise in temperature.

Figure 5 gives the length of the fission gas plenum for the same fuel rod as a function of wall thickness for a burnup of 68,000 MWd/ton (axial

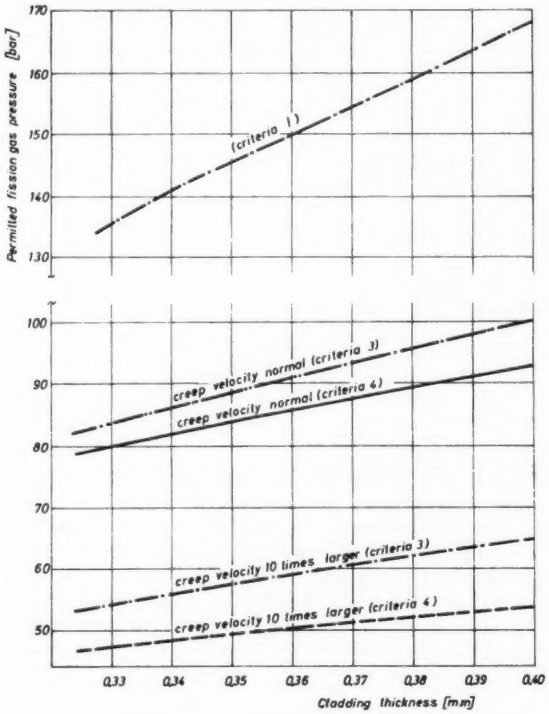

Fig. 4. Permitted fission gas pressure for a X8-Cr-Ni-Mo-V-Nb-1613 cladding at 700°C.

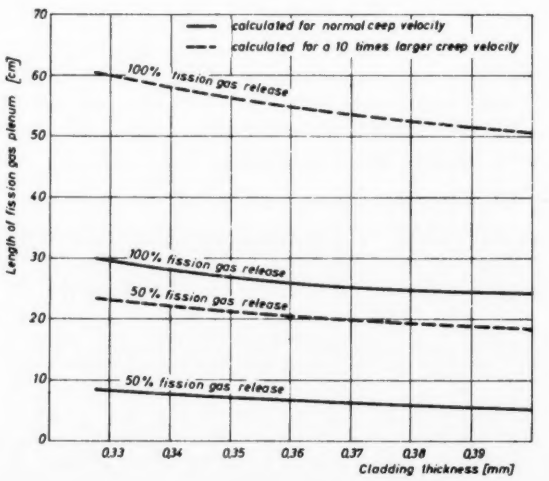

Fig. 5. Length of fission gas plenum as a function of wall thickness.

average) if 50 or 100% of the produced fission gas is liberated from the fuel, and under the assumption of normal and tenfold creep rates.[10] Even under this wide range of input variables, one stays within a reasonable range of plenum lengths between 5 and 90 cm.

In conclusion: For an austenitic steel cladding, a maximum temperature around 700°C is reasonable, but a range of ±20°C is not forbidden by principal considerations. The temperature optimization is not yet possible and has to include the variables burnup, wall thickness (breeding ratio and critical mass), and plenum length (pressure drop). Other materials, like vanadium alloys,[14] are being contemplated, but no quantitative argument can be given today.

Normal Cooling Conditions

To reach this cladding temperature, a number of cooling parameters must be established, of which the actual heat transfer coefficient is of minor importance. This can be seen[10] from Fig. 6, where, for a number of cases, the inlet temperature ϕ_1 and the inlet-outlet temperature difference $\Delta\phi$ have been varied. The maximum inside cladding temperature (top line) is given by the several parameters as follows:

1. The hottest point of the cladding for sodium cooling is practically always at the core outlet. Therefore, the total $\Delta\phi$ must be taken into account.

2. The radial blanket in this case is cooled in parallel flow to the core. At the end of the blanket life, it produces about 10% of the thermal power. If the blanket is fresh, this additional power has to be produced by the core and raises its temperature.

3. In the peripheral channels of each subassembly, the sodium flow normally is different from that in the inner channels for geometrical reasons. For a given average outlet temperature, temperatures in local channels, therefore, will differ from the average.

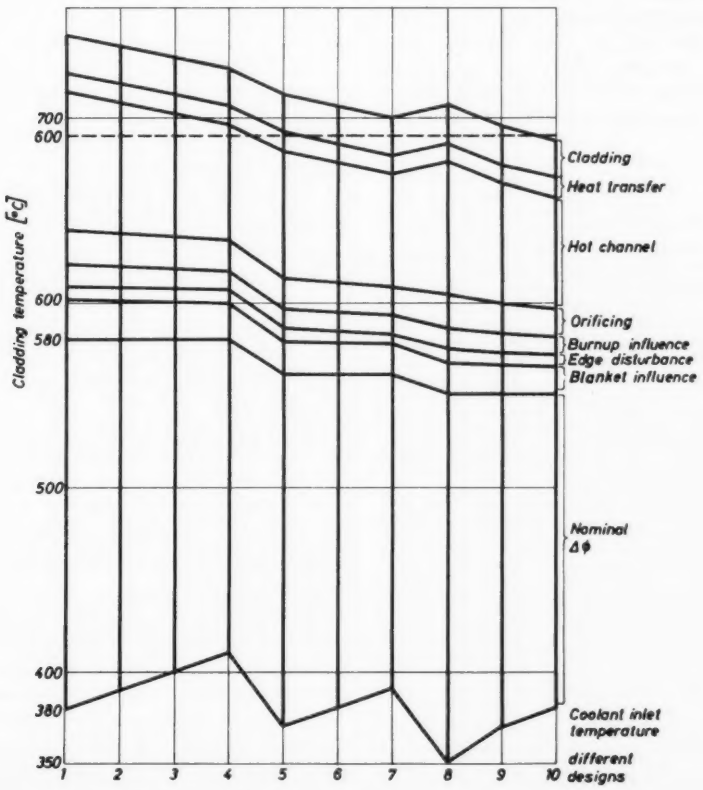

Fig. 6. Temperature buildup for different sodium-cooled designs.

4. Depending on the number of refueling cycles per fuel lifetime, there is a certain burnup swing. For our example, the cycle number was 3.

5. For the case of a radial power gradient, the inner side subchannels are at larger power than those on the peripheral side because of the small amount of mixing in a subassembly. This results in temperature gradients, especially with the low mixing ability of the grid spacers as used in all sodium-cooled core designs for reasons of axial rod movement. Optimum orificing of the coolant flow will provide for the same maximum outlet temperatures.

6. The hot channel factors are of particular importance. Table I shows the values used for this example.[10] Here a semi-statistical approach has been used, distinguishing between statistical and systematical errors. In addition to this, the connection between confidence level and error distribution, and their effect on parallel flow channels, must be considered.[15] A study[16] shows that a complete statistical approach for a confidence level of 97.7% leads to a similar overall hot channel factor as in Table I. Since the hot channel temperature rise is proportional to the nominal temperature rise, the largest cladding temperatures are obtained for the largest $\Delta\phi$ as can be seen by comparing case 8 to case 7, where a decrease in inlet temperature and a smaller decrease in outlet temperature still may result in an increase of cladding temperature.

TABLE I

Hot Channel Factors (semi-statistical) for Na-Cooled Cores

	Tolerances and Uncertainties	Coolant Temp Rise	Heat Transfer Coefficient	Rod Power
	Dimensions	1.05	1.02	1.00
	Fuel density	1.02	1.00	1.02
	Fuel composition	1.02	1.00	1.02
	Flow distribution	1.05	1.03	1.00
	Local flux perturbations	1.05	1.00	1.05
Statistical factor	(97.7% confidence level)	1.093	1.036	1.057
	Macroscopic flux distribution	1.07	1.00	1.07
	Heat transfer	1.00	1.15	1.00
	Thermal power evaluation	1.06	1.00	1.06
	Miscellaneous	1.03	1.00	1.03
Absolute factor		1.17	1.15	1.13
Summary		1.28	1.19	1.20

7. The temperature difference coolant to cladding is comparatively small. Normally it may be calculated according to Dwyer[17,43]; some references show this to be too optimistic,[12,13,18] but this is of really minor influence.

8. The temperature difference across the cladding finally defines the working temperature with respect to stresses.

Hence, we may formulate the following relation for the maximum clad temperature as a function of burnup Bu and plenum length h_p (neglecting other parameters).

$$T_{max}(Bu, h_p) = \phi_1 + \Delta\phi \cdot H + \text{const} \qquad (2)$$

or

$$Bu = Bu(h_p, \phi_1, \Delta\phi \cdot H), \qquad (3)$$

where $\Delta\phi \cdot H$ is the coolant temperature rise of the hot channel (H: hot channel factor). There is no explicit formulation of Eq. (3) available (except for the model for ratcheting), and any optimization has to consider the coupling to the whole system of the variables of Fig. 1. In Fig. 7, the range of different designs with respect to ϕ_1 and $\Delta\phi$ is shown.[10,19-21]

According to our own work,[5] the optimum $\Delta\phi$ should be around 180°C, but most designs have decided for lower values. By this, the first variable of the first column of Fig. 1 is defined.

We now turn to the next variables of the second column of Fig. 1. The rating r is given by

$$r = \frac{\bar{q}_r}{\pi d_r \rho_f e} \qquad (4)$$

It is the mentioned optimum between high rating (small d_r) and low fabrication costs (large d_r) that fixes this value. All other variables are of minor importance.

r = 0.7 to 1.0 MW/kg fissile (oxide fuel).

The breeding ratio BR is fixed by physics.

$$BR = BR(F_c, h_c, d_c, \rho_f) \qquad (5)$$

Low leakage cores are characterized by $h_c \approx d_c$, a large positive void, a large negative Doppler effect, and a large internal breeding ratio. Flattening will shift the system to external breeding without changing the overall BR too much. External breeding means large reactivity swings and capital charges on blanket plutonium. The better void effect is paid for by a worse Doppler coefficient. The most important economic influence of flattening is the increased fabrication cost by the increased number of fuel pins.

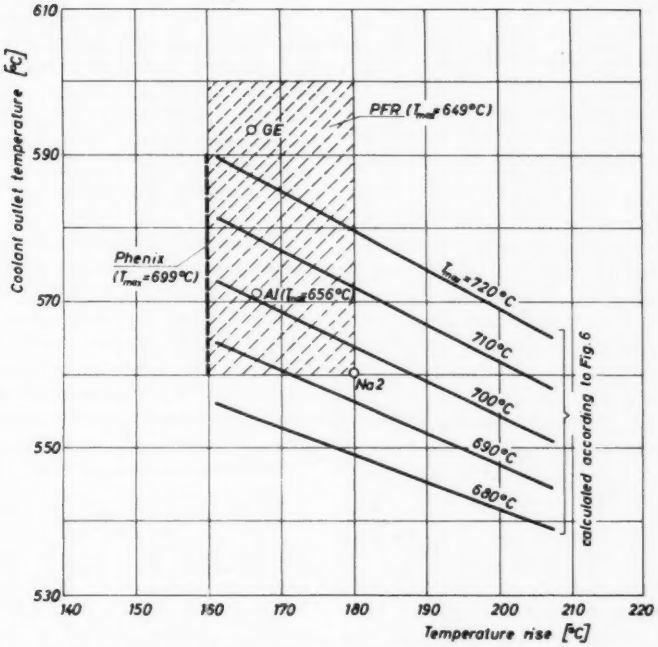

Fig. 7. Maximum clad temperature as a function of coolant outlet temperature and temperature rise.

A correct optimization of the *BR* in connection with that of h_c and d_c has not been made. Current designs[10,19,20] favor $h_c \approx 80$ to 100 cm, which in combination with a 45 to 50% coolant fraction leads to reasonable pressure drops on the order of 2.5 to 5.0 bars. For 1000 MW(e), this means $h_c/d_c \approx \frac{1}{3}$. Even in the US, where flat or otherwise leaky cores have been favored for some time,[23] the same tendency has developed.

Finally, the efficiency η of sodium-cooled reactors can be expressed in a simple way. As shown in Fig. 6, the sodium outlet temperature ϕ_2 has to be about 120°C below the maximum can temperature. Assuming two sodium systems, η can be calculated to the first order as a function of the sodium temperature and, therefore, as a function of cladding temperature as shown in Fig. 7. Efficiencies of 40 to 42% reflect the current status. This is clearly shown for different designs in Fig. 8.

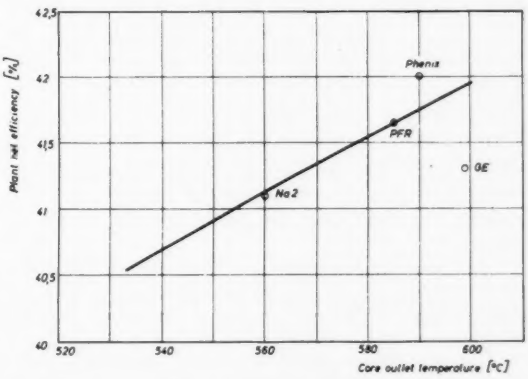

Fig. 8. Approximate plant net efficiency as a function of sodium core outlet temperature.

Concluding this paragraph on normal core cooling with sodium, the main areas for further improvement are as follows:

1. Better power flattening to get larger average rod powers
2. Better coolant mixing in subassemblies to allow for larger efficiencies at the same can temperature
3. Better understanding of the cost of tight tolerances vs gain in hot channel factors, thus giving larger efficiencies
4. Optimization of the h_c/d_c ratio
5. Finally, as a clearly evident statement, development of a fuel element with high burnup, high density fuel, low absorption, low creep, and high-temperature cladding.

Sodium Boiling

The normal operating temperatures are 350°C below the boiling point of sodium. Under accidental conditions, however, boiling may start and lead to dangerous consequences in connection with the positive void coefficient. We have to distinguish two cases:

1. Boiling over the whole Core Region: This could be caused by a loss of coolant flow *combined* with a complete safety system failure and is therefore *very improbable*. In this case, the axial liquid ejection rate and the radial spreading of the boiling zone determine the reactivity input rate and, from this, the destructive energy of the following excursion. Reactivity rates on the order of 50$/sec, and mechanical energies on the order of 1000 MWsec, have been calculated by the various groups. Most groups do not consider this accident to be beyond the design basis accident.[55]

2. Boiling in a Single Subchannel or Subassembly: This could be caused simply by a local flow blockage, fuel element swelling, or can failure, and is *much more probable* than (1). In this case, the reactivity input is negligible and the *ejection rate therefore unimportant*. But pressure pulses by the sudden flashing of superheated liquid or even more by recondensation of vapor bubbles may affect the neighboring channels or subassemblies and result in a *fast propagation*[a] of the failure. Finally, a fast reactivity input is again created and the consequences are similar to case (1).

I am not going to discuss the safety aspects of this which depend mainly on the reliability of several engineered safeguards. Rather, I shall restrict myself to the aspects of sodium boiling itself.

In connection with cases (1) and (2), ejection rate, liquid superheat, and recondensation must be considered.

Ejection Rate. Quite a number of models have been proposed.[56-59,61] Today the best fit to measurements[49,60] is given by the BLOW-code.[50,54] In the first phase, a small bubble forms in the superheated liquid. Because of superheat, normally just one single bubble develops at a time in sodium. The second phase describes the growth of a spherical bubble, followed by a third phase with the growth of a new cylindrical bubble, fed by evaporation of a thin liquid film on the heated channel surface. It is this process, particularly, that distinguishes the BLOW-model from the earlier ones. In the final phase, the liquid film dries out, but this may be overrun by a return flow of the liquid into the channel. The model still needs more refinement for the first instants of nucleation and bubble growth, but in general, the ejection process does not present any more principal theoretical difficulties.

Liquid Superheat. Liquid superheat is an important input parameter for ejection codes; it is not well understood. The overpressure in a bubble of radius r is according to the following well-known relation: $\Delta p = 2\sigma/r$, where σ is surface tension. This corresponds to a certain rise in the saturation temperature. For the initiation of boiling, therefore, a nucleus is needed, which is either in the liquid or at the surface. The nature of these nuclei is still unknown.

- Active cavities at the surface should be destroyed by the wetting action and chemical aggressivity of the long-time operation in the liquid phase.[54] Holtz[53] has formulated a phenomenological model on the activity of cavities, but the physicochemical nature of the activation and formation of the nuclei is by no means clear. Hoffman et al.[51,52] have already used deep cavities for nucleation, and in experiments of Schultheiss,[65] very simple cavities stayed active even when completely filled with sodium.

- Nucleation by irradiation, according to Claxton,[62] must also be excluded.

- Spontaneous (statistical) nucleation in the range of interest is very improbable.

- The effect of dissolved gases is too small.[63]

- Only entrained gas bubbles, possible in designs with free surface pumps, may have a considerable effect. They constitute the most efficient safety measure against superheat. Effective bubble nuclei still are so small that reactivity disturbances can be avoided.

On the other hand, the measurements, although widely spread, show relatively low superheats under reactor conditions. Values on the order of 25 to 50°C are to be expected. So, from the practical point of view, superheat could be handled, whereas the theoretical insight into the nucleation phenomena still is rather poor. More effort on experiments with controlled physicochemical conditions of liquid and surfaces is needed.

Recondensation. The reactor system implies heated core channels, unheated axial blanket channels, and the sodium pool above and below the core. Vapor bubbles from the core region reach

[a] This is to be distinguished from the slow failure propagation by fuel element melting as in the Enrico Fermi incident.

cooler zones and recondense quickly. Moreover, measurements show a periodic flow reversal with sodium returning to the heated region. Basically, this is a typical feature of the unstable boiling of the liquid metals.[51,52] Peppler[84] demonstrated this for channels of 50-cm heated length, unheated portions above and below, and wall heat fluxes of several hundred W/cm^2; i.e., typical reactor conditions.

The condensation of bubbles is accompanied by water-hammer-type pressure pulses,[63] very narrow (≈1 ms) peaks with up to 30 atm. They depend on the channel geometry and the flow and temperature profile as well. There is some evidence that with increasing superheat and, therefore, more vigorous ejection, the reverse flow and recondensation phenomena are enlarged.

Because of their short duration, the energy of even large recondensation pressure peaks is low. Therefore they probably will not cause subassembly destruction and fast failure propagation. Because recondensation and its effect on core structure depends on the complicated geometry, a refined theoretical analysis is impossible at present. For the final proof, experiments in multirod and even multi-subassembly geometry are needed.

STEAM

General Cooling Conditions

The main reason for steam cooling is the possibility of a direct cycle with components more or less based on light-water technology.[1,3,31,41,45-48] I will not discuss this aspect, but will confine myself strictly to the core. Here, steam is not as good a coolant as sodium for two reasons: It is a gas and it moderates.

To not exceed a given clad temperature with gas, we now have to consider a not-negligible temperature difference coolant to wall; in other words, we have to worry about heat transfer. Figure 9 shows the typical buildup of temperatures for steam and helium compared to one of the sodium cases of Fig. 6. The mixing and similar influences are of the same order, whereas $T - \phi$, the clad-to-coolant temperature difference, has become much more important, (e). Also the hot channel influences on heat transfer become important, (f). The maximum clad temperature then no longer occurs at the channel exit. Therefore, the hot spot is in the region of larger heat fluxes leading to an increase of ΔT in the cladding, (g). For saturated steam at the core inlet (direct Loeffler cycle), the inlet temperature is determined by the steam pressure.

All these additional conditions reduce the degree of freedom for the choice of variables. In general, we have to restrain the following three parameters within a certain range of values: $T_{clad} - \phi$, $\Delta\phi$, and Δp, by three variables: G (flow rate), F(channel cross section), and h_c (core height).

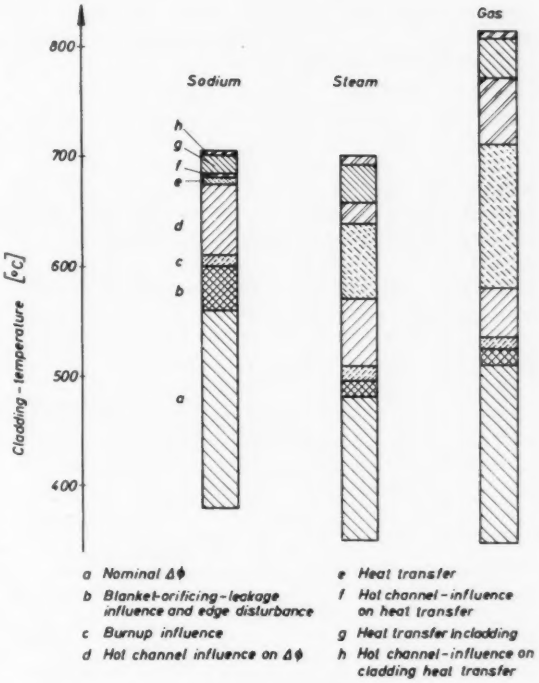

Fig. 9. Cladding temperature buildup at location of hot spot temperature for gas and steam cooling compared to sodium cooling.

For sodium, the first parameter ($T_{clad} - \phi$) is unimportant, therefore one of the variables (for instance h_c or F) is independent and may be chosen for maximum breeding, minimum fuel cycle costs, or low void effects. For gaseous coolants, there is a strict interconnection and no free variable. Also, the resulting Δp is of greater importance, whereas for sodium, the pressure drop is limited by some structural conditions not strictly defined; for gaseous coolants, it determines the pumping power and, therefore, the efficiency to a very great extent.

Considering the moderating qualities of steam, we get another condition on our variables: The channel area F and, therefore, the steam fraction in the core, has to be kept at a minimum. This is given by design and tolerance levels in the subassembly at a coolant volume fraction in the range of 30% or a pitch-to-diameter ratio of 1.15 (in the

case of a pin diameter of 7 mm). Therefore, for steam-cooled fast reactors, there is a unique interconnection between

$$\frac{T_{clad} - \phi}{\Delta\phi} \quad \text{and} \quad \frac{G}{h_c} \quad ,$$

whereas F and, as a consequence, Δp are now fixed by other reasons. A certain desired steam exit temperature will result in a certain core height or a certain h_c/d_c, and vice versa. We shall point out this strict intercoupling later when looking at the numerical results.

The fuel element is determined by the low coolant fraction and the heat transfer requirements. Grid spacers would raise a large pressure drop. The most favorable solutions here are spiral wire or spiral fin spacers (Fig. 10). The spiral spacers will also help in coolant mixing between neighboring channels (but not across the whole subassembly).[24-26] This mixing will peel off about 4.5%/cm of the coolant mass flow and reduce the hot channel factor for $\Delta\phi$ from 1.60 to about 1.31.

An improvement in heat transfer can be obtained by turbulence promoters, especially of the boundary-layer type. Measurements on their effectiveness have been made for tube[27] and rod

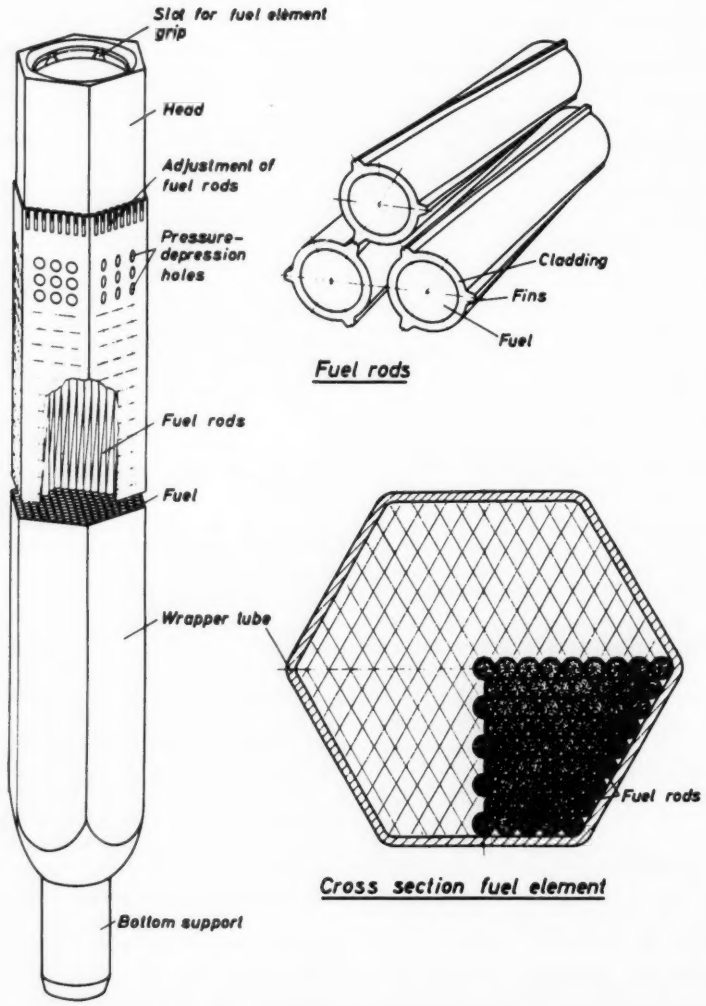

Fig. 10. Fuel element of steam-cooled reactor.

geometries.[28] Exact values have not yet been established for combinations of spiral spacers of different pitch and boundary-layer turbulence promoters. The best data for turbulence promoters, at present, are: rise of friction coefficient by a factor of 5 (taking into account the corrosion abrasion to the steam flow), and rise of heat transfer coefficient by a factor of 2.

This allows for a substantial efficiency increase. But, since T_{clad} - ϕ, $\Delta\phi$, G, and h_c are strictly coupled because of the fixed and small flow area, the use of turbulence promoters must result in a smaller h_c. Physically, this means that to make use of the higher heat transfer numbers with turbulence promoters, one has to overcome the larger friction and Δp by lower core heights. So, for steam, the helpful use of turbulence promoters leads to very flat cores with low internal breeding and large numbers of fuel pins. This gives an increase in fuel cycle costs.

The coolant pressure is another important choice. The Karlsruhe work[2] shows a decrease of capital costs with increasing pressure, whereas fuel cycle costs first decrease (gain in efficiency with pressure), then again increase (loss in breeding ratio). Therefore a minimum of power generating costs is around pressures of 150 bars. Others[1,29] have not found as distinct a pressure dependence or favor pressures quite as low.[46] Some proposals, therefore, use supercritical pressures.[31,41,48] They may have low capital costs, but cannot be considered to be breeders.

Maximum Clad Temperature

In considering all cooling conditions, we finally have to fix the maximum clad temperature. While the internal fission gas pressure defines the design temperatures with respect to cyclic creep deformations for steam, it is the external coolant pressure with respect to creep collapse for gas and steam cooling. Creep collapse is the creep-induced oval deformation of the clad tube and is most dangerous for the fresh fuel rod. Later, the built-up fission gas pressure will compensate for this effect.

A theory on creep collapse of empty tubes has been developed by Hoff.[32] Experimental results[33] check with this reasonably well. Figure 11 gives results for 7-mm-o.d. tubes of Inconel-625 and Incoloy-800. At cladding temperatures of 700°C, which are necessary for a reasonable efficiency, even Inconel-625 tubes must be considerably inflated with gas to withstand the coolant pressure.

The fuel will definitely give a certain support to the clad. But with fuel surface temperatures on the order of 800°C, most of the fuel is in the plastic range ($T > 900$°C). The actual strength of support is unknown and certainly limited, and as of now, no experimental information is available.

Therefore, a conservative design must use Inconel-625, a "stronger cladding" with respect to creep collapse. Inconel-625 is a relatively strong neutron absorber because of its content of nickel and molybdenum (Table II). A more opti-

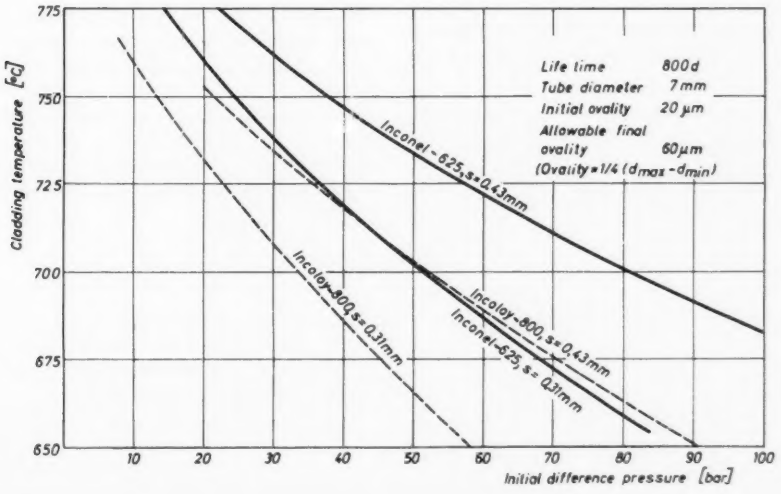

Fig. 11. Cladding temperature as function of the initial pressure difference outside-inside of the cladding (at hot conditions) for Incoloy-800 and Inconel-625 as a result of creep collapse calculations.

TABLE II

Composition and Absorption Cross Sections of Several Structural Materials for Steam-Cooled Fast Reactors

Alloys	Maximum Content (wt%)										Absorption Cross Section for Fast Neutrons of 100 keV (mb)
	C	Fe	Cr	Al	Ti	Si	Ni	Mn	Mo	Nb	
Incoloy-800	0.1	38.2	23	0.6	0.6	1.0	35	1.5			8.73
Inconel-625	0.1	5.0	23	0.4	0.4	0.5	56	0.5	10	4.1	17.06

mistic designer willl use a "weaker cladding," either supported by the fuel or by artificial gas pressure ("blow up") of the Incoloy type with lower neutron absorption. The alloy, Sandvik 12-RX-72, finally has a composition similar to Incoloy-800, but may allow for higher temperatures. So even turbulence promoters may be unnecessary, thus allowing[34] for a larger h_c.

Summary of Steam-Cooled Reactor Problems

Summarizing the problems of steam-cooled fast reactor core design, we have the following situation:

1. Clad temperatures are determined by creep collapse. An economic design with good breeding must rely on fuel support, the amount of which is unknown, or extra internal pressurization with all the operational implications is necessary.
2. If strong clads like Inconel-625 are necessary, a loss in breeding ratio is unavoidable.
3. The moderating qualities of steam require narrow coolant channels with the implication of higher hot channel factors and the effects of structural swelling.
4. For good heat transfer at low pumping power, turbulence promoters are necessary. Together with the preceding condition, this requires flat cores leading to higher fuel cycle costs.

The most recent analysis of the potential of steam cooling has been made within the ENEA working group.[34] Table III summarizes the most important results.

The technical and economic ground rules are outlined in the original document. The Pu-α values are based on recent ORNL data determined by Gwin et al. Using the more pessimistic values of Schomberg,[35,36] a small breeding decrease is to be expected.

The direct capital costs have been estimated to be 125 to 130$/kW for a 1000-MW plant. On this basis, the power generating costs of Table III have been calculated.

The results of the USAEC evaluation of alternate coolant fast breeder reactors,[44] based on the older Karlsruhe D1 design and Babcock & Wilcox proposals, are somewhat less favorable. The power generating costs for sodium cooling are in the range of those in Table III, but breeding ratios are larger.

The main problem of the steam-cooled fast reactor is the fuel element behavior. Statistical testing in a fast flux and steam environment is essential. Since test reactors are neither available nor under construction, it seems difficult to keep up in time scale with the more advanced sodium line. Since the economic potential of steam cooling does not exceed the potential of sodium (and is inferior to sodium-cooled reactors with carbide fuel) it is still under consideration if it is justified to spend the necessary development effort.

TABLE III

Cladding	h_c/d_c	BR	η	Fuel Cycle Costs (mills/kWh)	Total Power Generating Costs (mills/kWh)	Degree of Optimism
Inconel-625, 700°	0.198[a]	1.12	0.371	1.32	3.82	conservative
Incoloy-800, 700°	0.198[a]	1.19	0.371	1.10	3.60	optimistic
Sandvik, 12-RX-72, 735°	0.347[b]	1.21	0.362	1.00	3.50	more optimistic

[a]Clad with turbulence promoters.
[b]Smooth clad (ENEA Reference Design).

GAS

The first proposal on a helium-cooled fast breeder was made in 1961 by the Karlsruhe group.[42] More detailed work was published at the 1963 Argonne Conference.[38] During this time, General Atomic started their own work,[22,37,39,40] whereas the Karlsruhe interest shifted to sodium and steam cooling. Later, Swedish,[1] German,[30] and UKAEA groups again took a strong interest.

Conditions for gas cooling of rod-type metal-clad fuel elements are similar to steam cooling with a few very important exceptions:

1. He or CO_2 moderate only slightly; therefore, the breeding ratio is larger and the void coefficient less positive.
2. The channel cross section or coolant fraction can be varied; therefore, there is no strict coupling of the h_c/d_c ratio to $\Delta\phi$. This again allows a broader range of core geometries to choose low fuel cycle costs.
3. Future designs even may use a vented fuel element,[66] where the internal pressure is equal to the coolant pressure. This solves the problem of creep collapse, which is otherwise the same. Hence, cladding temperatures of 770°C seem to be possible even with a 316 SS material.

On the other hand, the indirect cycle is indicated with a possible penalty in capital costs. While reloading of the steam-cooled core is simple in the flooded condition, complicated loading machines are required.

The USAEC, as well as the ENEA, has also evaluated gas-cooled reactors.[67,68] These studies were based on work performed by GGA, AB Atomenergi-Sweden, UKAEA, Belgonucléaire, and GfK-Karlsruhe.

Table IV shows some of the results of the ENEA study.

By employing a direct cycle with a gas turbine, the capital costs may be lowered to about 120$/kW or power costs of 3.1 mills/kWh. This agrees with the US study in the range of accuracy.

On this basis there seems to be a small cost advantage for the gas cooling. However, it has been pointed out by the ENEA working group that the latest steam studies, aimed at a prototype, are already more refined than the gas studies. For instance, our group has recalculated the second reactor of Table IV with hot channel factors consistent with those in use for steam-cooled reactors.[69] It turned out that for the same clad hot spot temperature of 769°C, the nominal temperatures had to be much lower. This caused the efficiency to drop to 36%.[b] Without putting too much importance to this, it points out the strong dependence on the hidden assumptions. Therefore the estimated power costs can only define a certain range.

I should conclude that, at present, only the potential of the direct-cycle solution might justify the development of a gas-cooled fast reactor besides the running effort on the sodium line. Therefore, the feasibility of this concept should be evaluated more carefully.

CONCLUSIONS

1. *Sodium* is the coolant with the best cooling capacity, and by far the most experience exists on this material. The remaining problems of economic components and reliable engineered safeguards are a matter of experience. Therefore, sodium reactors should be pushed forward in any economy, because only then can this experience be gained. In this connection, I personally believe more in experience of actual proto-

[b] The same result has been achieved by the USAEC Alternate Coolant Task Force Study (Ref. 68, p. 1.9) if the hot channel calculation is based on an "overpower" of ~10% (due to neutron flux distortion by control rods and uncertainties in the power measurement).

TABLE IV

Fuel Type	Cladding	*BR*	η	Fuel Cycle Costs (mills/kWh)	Capital Costs ($/kW)	Power Costs (mills/kWh)
Oxide, sealed pin	Sandvik, 12-R-72-HV, 730°C	1.50	0.40	0.76	138	3.43
Oxide, vented fuel pin	Stainless steel 316, 769°C	1.51	0.41	0.76	138	3.43
Oxide, coated particles	Silicon carbide	1.27	0.42	0.91	132	3.48
Comparison with ENEA Steam-Cooled Reference Design						
Oxide, sealed pin	Sandvik, 12-RX-72, 735°C	1.21	0.362	1.00	126.4	3.50

types over test facilities. It is only in the prototype that the real problems can be defined and solved.

2. *Steam* has suffered some in prospect, although a gain in power cost can be reached compared to light-water reactors. The fuel element is still an open problem. The use of steam, therefore, is still open if cost-effectiveness considerations can justify an expensive development program.

3. *Gas* has a good future outlook with respect to breeding, safety, and costs, but because not much is known about it at present, it should be considered a long-range undertaking.

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Variational Techniques as a Method for Multidimensional Reactor Calculations

By Myrna L. Steele*

To fulfill the potential of the reactor as an economic power source, designers are turning to larger reactors and systems. Naturally, as the reactor and associated systems grow in physical size, they also grow in complexity. This complexity, in turn, makes more urgent the need for the reactor physicist to describe concisely and accurately the processes occurring in-core which affect reactor behavior and thereby the economics of the reactor.

Systems complexity and the increasing need for accuracy underlie the almost inescapable conclusion that three-dimensional calculations are going to be the requirement, rather than a desirable condition, for the liquid-metal-cooled fast breeder reactors (LMFBR's) as well as for the larger light-water reactors.

To date, three-dimensional calculations have been limited in practice to Monte Carlo methods, and these calculations generally require computing times on the largest and fastest digital computers available which are prohibitive for routine design work and totally infeasible for survey or preliminary design work.

One mathematical tool that has received considerable attention in the literature over the last 10 years, and particularly over the last 5 years, is the variational principle, or the so-called variational-synthesis technique.

Summarized below, for the period 1966 through June 1969, are the papers and reports on applications of variational-synthesis techniques to the neutron-diffusion and -transport equations arising in reactor-physics calculations. The purpose of these summaries of the literature is twofold: to present for easy reference the information generated in this 3-year period on the development of these variational-synthesis techniques and to serve as an informative survey for those who are concerned with developments in reactor physics but who are not, themselves, doing the development work.

Note that, in instances of two or more very similar papers by the same author, I have summarized the paper I consider to be the most readily available.

GENERAL DEFINITIONS AND METHODOLOGY

All synthesis techniques for solving neutron-diffusion and -transport space-, space-time-, or space-energy-dependent equations have some characteristics in common, e.g., base functions that are not necessarily analytic functions but are specifically fitted to a given physical problem, and the combining of base functions to arrive at an approximate solution to a given problem. A generalized outline for the application of variational-synthesis techniques to a reactor (or any boundary value) problem is as follows:

1. The functional is defined and the combining (or expansion) coefficients are chosen so that the functional is stationary about some eigenvalue or set of eigenvalues.

2. Base functions are determined from preliminary calculations, and the trial function is selected.

3. The combining (or mixing) coefficients within the trial function are then determined from variational principles to yield the synthesis equations.

The functional is often an approximation of some physically significant quantity. The stationarity property of the functional then means that first-order errors in the estimation of the minimum value of an eigenvalue of an equation lead only to second-order errors in the value of the functional.

Henry[1] summarizes the time-synthesis and space-time-synthesis techniques for solution to the few-

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group neutron-diffusion equations arising in reactor-physics calculations. He characterizes the time-synthesis problems as those in which flux shapes are assumed over the entire core and the space-time-synthesis problems as those in which flux shapes are assumed over only some portions of the core. He describes in some detail the following time-synthesis methods: general, modal flux shape, Helmholz mode, collapsed group, adiabatic approximation, and point kinetics.[1]

spaces, into a real number. Another way of explaining the purpose of the functional is: a functional reduces an infinite dimensional space to a finite dimensional subspace by imposing bounds on the subspace and examining the error introduced by the use of this bounded subspace.

Froehlich[3] recently discussed conditions that assure success in applying synthesis techniques to the various physical problems arising in reactor analysis. These are summarized in Table 1.

Table 1 Theory and Anomalies for Variational Flux-Synthesis Techniques in a Discrete Diffusion-Theory Model[3]

Problem type		Coarse-mesh techniques		Bettis-type blending	
		Rebalancing	Galerkin	Galerkin	Adjoint
One group	Theory	Complete	Complete	Largest positive eigenvalue exists	Largest positive eigenvalue exists
	Experience	Good	Good	Good	Good
Down scattering only, no group collapsing	Theory	Complete	Complete	?	?
	Experience	Good	Good	Good	Good
Down scattering only, with group collapsing	Theory	Complete	Counter example; complete with simple additional condition	Counter example	Counter example
	Experience		Not always good	Limited	Bad
Up and down scattering, no group collapsing	Theory	Complete	Counter example; complete with simple additional condition	Counter example	Counter example
	Experience	Good	Limited	Limited	Limited
Up and down scattering, with group collapsing	Theory	Complete	Counter example; complete with simple additional condition	Counter example	Counter example
	Experience	Good	Limited	Limited	Limited

Wachspress[2] uses the stationarity property of the functional to show that first-order errors in minimum-eigenvalue estimation lead to second-order errors for the value of the functional for discrete multigroup equations. He reviews the early approaches to synthesis-type reactor calculations and traces synthesis development from a region synthesis to the use of variational principles and finally to functionals that admit discontinuous trial functions.

In the terms used by Wachspress,[2] the functional serves the purpose of mapping each pair of direct and adjoint functions, which lies within specified function

GENERAL MATHEMATICAL CHARACTERISTICS OF VARIATIONAL-SYNTHESIS METHODS

Kaplan and Davis[4] cite the use of variational principles as applied to transport problems by Vladmirov, Selengut, Pomraning and Clark, and Davis and show how these different formulations are interrelated. They start with one of Vladmirov's principles which has the even-parity second-order form of the transport equation as its Euler equations, i.e., $d_1^G = F_1^G(\Psi) = \min_{\eta} F_1^G(\eta)$ and $F_1^G(\eta)$ is defined as:

$$F_1^G(\eta) = \int_R \left\{ \frac{1}{2}\langle \Omega \cdot \nabla\eta, K_U\, \Omega \cdot \nabla\eta \rangle + \frac{1}{2}\langle \eta, g_G \eta \rangle \right.$$

$$- \langle \Omega \cdot \nabla\eta, K_U S_U \rangle - \langle \eta, S_G \rangle + \frac{1}{4}\,[\langle S_G, K_G S_G \rangle$$

$$\left. + \langle S_U, K_U S_U \rangle] \right\} d\mathbf{r}$$

$$+ \frac{1}{2} \oint_S \left\{ - \int_{\Omega\cdot\mathbf{n}<0} (\Omega \cdot \mathbf{n})\eta^2\, d\Omega \right.$$

$$\left. + \int_{\Omega\cdot\mathbf{n}>0} (\Omega \cdot \mathbf{n})\eta^2\, d\Omega \right\} dS$$

They then apply involutory and canonical transformations to obtain equations similar to those derived by Selengut.

Stacey[5] starts with an inhomogeneous linear equation in n-independent variables of the type $H(x_1 \ldots x_n) - f(x_1 \ldots x_n) = 0$ from which he develops an approximate set of equations for linear systems. He refers to this approximation method as a modal-expansion method. Stacey then discusses other analytical techniques that are of use to the reactor physicist such as harmonic analysis, polynomial expansions, semidirect variational methods, and weighted-residual methods. He also lists generic formulations for error criteria.

Pomraning[6] derives a generalized Roussopoulos principle for inhomogeneous equations of the type $H(x)\Psi(x) = S(x)$ and shows that the functionals already in use are derivable from it. He uses the Lagrange multiplier technique to derive the variational principle and then derives a generalized Schwinger principle and two Nth-order principles for estimating an arbitrary functional of a solution to an inhomogeneous equation. He then gives an example of how the Lagrange multiplier technique can be used to derive variational principles that permit use of discontinuous trial functions.

Wachspress[7] uses the variational principle to determine the eigenvalues of a system of first-order differential equations of the Sturm-Liouville type, using discontinuous trial functions, i.e.,

$$-\frac{d}{dx} D(x) \frac{d\phi(x)}{dx} + [A(x) - \lambda]\ \phi(x) = 0$$

$$\phi(0) = \phi(X) = 0$$

The free parameters in the trial function are determined from the stationarity equations,

$$\frac{\partial F}{\partial a_{mn}} = \frac{\partial F}{\partial b_{mn}} = \frac{\partial F}{\partial c_{mn}} = 0$$

for $m = 1, 2, \ldots M$ and $n = 1, 2, \ldots N$. He uses finite difference techniques to obtain the trial functions and a mesh spacing of one to solve the base problem. He gives numerical solutions to an eigenvalue problem, a materials-region problem, and a flux-tilt problem. Numerical comparisons illustrate the differences between standard variational techniques and procedures admitting discontinuous trial functions for one-dimensional calculations.

Both Davis[8] and Toivanen[9] have utilized variational techniques for derivation of boundary conditions for the P_1 approximation of the spherical-harmonics method for solving the stationary-monoenergetic neutron-transport equation.

Toivanen uses the integrodifferential form of the energy-independent transport equation to establish the self-adjoint form:

$$\Omega \cdot \text{grad}\, \phi(\mathbf{r},\Omega) + \Sigma_t(\mathbf{r})\, \phi(\mathbf{r},t) = \frac{1}{4\pi} C(\mathbf{r})\, \Sigma_t(\mathbf{r})$$

$$\times \int_\Omega f(\mathbf{r},\mu_0)\, \phi(\mathbf{r},\Omega')\, d\Omega' + S(\mathbf{r},\Omega)$$

where $\phi(\mathbf{r},\Omega)$ = directional neutron flux
$S(\mathbf{r},\Omega)$ = external directional source
$C(\mathbf{r})$ = mean number of secondary neutrons per collision
$\Sigma_t(\mathbf{r})$ = total macroscopic cross section

Because of the first-order differential operator in the leakage term of the Boltzmann equation, the transport operator does not belong to the class of self-adjoint operators. In order to remove this nonself-adjointness, Toivanen invokes standard mathematical procedures for separating the equation into odd and even forms (with respect to the cosine of the neutron-scattering angle); the integrodifferential equation, after separation, becomes two ordinary differential equations. Normalization to one of the neutron-collision kernels causes the value of one of the differential equations to be zero.

Next Toivanen considers the eigenvalues and eigenfunctions of the odd homogeneous Fredholm integral equation:

$$\frac{1}{4\pi} C(\mathbf{r}) \int_\Omega f^-(\mathbf{r},\, \mu_0)\, \phi^-(\mathbf{r},\Omega')\, d\Omega' = \lambda \phi^-(\mathbf{r},\Omega)$$

By expanding $f^-(\mathbf{r}, \mu_0)$ into an odd Legendre polynomial series, utilizing the addition theorem of Legendre polynomials and the orthogonality property of spherical harmonics of different orders, he arrives at expressions for the eigenvalues and then derives a

self-adjoint equation for the even portion of the directional flux.

After establishing the self-adjointness of the even part of the total directional flux, Toivanen sets up a variational principle by formulating a Lagrangian whose Euler-Lagrange equation is equivalent to the even part of the flux. He points out that the reason that the functional (of the variational principle) can be a Lagrangian is that the expressions for the flux and variational functional are equivalent; this, in turn, satisfies the stationarity requirements of the transport theory. Thus one obtains not only the neutron-field equations for the even part of the flux expression but also the associated continuity and boundary conditions.

Toivanen then uses this newly derived self-adjoint variational form to arrive at boundary conditions for the P_1 approximation of the spherical-harmonics method. The derivation of the boundary conditions is based on a semidirect variational method consisting of varying the Lagrangian after substituting in the trial function and integrating this trial function over the angle variable. For example, assuming a trial function

$$\phi^+(\mathbf{r},\Omega) = \frac{1}{4\pi}\phi^+(\mathbf{r})$$

inserting this into the Lagrangian, and integrating over the angle variables yield a reduced Lagrangian:

$$F[\phi^+(\mathbf{r})] = \frac{1}{4\pi}\int_G \left\{\left[\frac{1}{3\Sigma_t}\frac{1}{1-C\langle\mu_0\rangle}\right|\text{grad }\phi^+(\mathbf{r})\right]^2 + (1-C)\Sigma_t\,\phi^{+2}(\mathbf{r}) - 2S^+(\mathbf{r})\,\phi^+(\mathbf{r})\Big\}\,d\mathbf{r} + \frac{1}{8\pi}\int_S \phi^{+2}(\mathbf{r}_s)\,d\mathbf{r}_s$$

Toivanen asserts that the solution to the problem of arbitrariness, which arises in the use of variational principles for deriving the P_N boundary and continuity condition, lies in the separation of the nonself-adjointness from the transport equation by using the coupled-directional-equation forms.[9]

Davis[8] also uses this coupled-directional-equation form of the stationary-monoenergetic transport equation, or, as he chooses to call it, the parity-equation form. However, Davis goes a little further and develops two stationary principles that have odd and even boundary conditions. He derives the approximate P_N boundary conditions for the odd P_N equations rather than for Toivanen's even equations.

Davis reviews and interprets the work of Pomraning and Clark with respect to the use of the variational principle for derivation of the P_N equations as a background to his work. He then writes the monoenergetic transport and adjoint equations for general geometry. Free-surface boundary conditions are imposed on both sets of equations. A functional is formulated and a variational principle applied. From this, Davis obtains an expression for the flux from the specified stationarity conditions. The trial functions postulated are in the form of a truncated spherical-harmonics expansion:

$$\psi(\mathbf{r},\Omega) = \frac{1}{4\pi}F_0(\mathbf{r}) \qquad \chi(\mathbf{r},\Omega) = \frac{3}{4\pi}\,\Omega\cdot F_1(\mathbf{r})$$

and adjoints:

$$\psi^*(\mathbf{r},\Omega) = \frac{1}{4\pi}F_0^*(\mathbf{r}) \qquad \chi(\mathbf{r},\Omega) = \frac{3}{4\pi}\,\Omega\cdot F_1^*(\mathbf{r})$$

The angular integrations required in the transport equations are performed. The resulting Euler equations are the direct and adjoint P_1 equations. This procedure is then expanded to higher order than P_1.

Davis[8] then derives the vacuum boundary conditions for the transport equation and explains the requirements on these boundary conditions.

Henry[10] applies the variational principles to the few-group diffusion-theory description of space-time neutron behavior in reactors. He starts with the continuous-energy equations for the P_1 approximation of the time-dependent transport equations and distinguishes between currents, fluxes, and precursors. The normal components are assumed continuous across any internal interface. In mathematical notation this becomes:

$\psi_p^0(\mathbf{r},E,t)$ = prompt-neutron flux

$\psi_p^1(\mathbf{r},E,t)$ = prompt-neutron current

$\psi_d^0(\mathbf{r},E,t)$ = delayed-neutron flux

$\psi_d^1(\mathbf{r},E,t)$ = delayed-neutron current

The total fluxes and currents are a combination of both the prompt and delayed components, or

$$\psi^0 = \psi_p^0 + \psi_d^0$$

$$\psi^1 = \psi_p^1 + \psi_d^1$$

which are the usual P_1 equations for the flux and current.

Henry then introduces definitions for the linear operators A, D^{-1}, M, and M_i to allow simplification of the transport equations. The adjoint operators are defined in accordance with the definitions for the direct operators, and the adjoint equations are similarly simplified.

Henry then selects a functional that is stationary with respect to the solutions for the simplified transport equations and their adjoints; the functional is chosen so that the domain of functions for which the functional is defined includes piecewise-continuous functions of position. The continuity requirements of the functions are that the functions be continuous in time, piecewise continuous in energy, and piecewise continuous over the region of the reactor. The boundary conditions at the outer surface are defined and stationarity requirements are developed to comply with these boundary conditions.

Henry's next step is to select trial functions for the synthesis procedure. These trial functions are selected from a subset of functions in the domain over which F is defined, i.e., the domain in which the functions are piecewise-continuous functions of position. He then effects a double partition of the reactor region into subregions which are contiguous and which cover the reactor. The functions and functionals are continuous within each subregion. By expressing the current-spectrum functions in terms of dyadics having a diagonal form and normalizing, Henry arrives at the following trial-function forms:

$$u_a = \sum_{m,r} \Gamma_r(\mathbf{r}) \psi^a_{m,r}(\mathbf{r},E,t)\, \phi^a_{m,r}(\mathbf{r},t) \qquad \alpha = p,d$$

$$v_a = \sum_{m,s} \Gamma_s(\mathbf{r})\, \tilde{\mathbf{J}}^a_{ms}(\mathbf{r},E,t) \cdot \mathbf{J}^a_{ms}(\mathbf{r},t) \qquad \alpha = p,d$$

$$C_i = \sum_r \Gamma_r(\mathbf{r})\, C^i_r(\mathbf{r},t) \qquad i = 1, 2, \ldots, I$$

where the tilde indicates a diagonal dyadic and other notation is standard. The corresponding adjoint trial functions are constructed in a similar manner.

Inserting these trial functions into the expressions for the functional and setting the variation of the functional equal to zero yield the equations for the flux, current, and precursors and for their associated internal boundary conditions. The adjoint equations are obtained in a like manner. All these equations are the very general time-dependent few-group P_1 equations, adjoints, and boundary conditions.

The derivation of the formally exact few-group equations involves taking the exact solutions to the transport equations, forming the angular and adjoint flux- and current-spectrum functions, and, then normalizing these spectrum functions. The direct and adjoint flux equations in matrix notation then become:

$$[(\mathbf{B}) + \nabla] \cdot (\tilde{\mathrm{D}}) \cdot [(\mathbf{B}) + \nabla](\phi) - (A)(\phi) + (M)(\phi) + \sum_i \lambda_i C^i(\chi_i) = (V)^{-1} \frac{\partial}{\partial t}(\phi)$$

and

$$[(\mathbf{B}^*)^T - \nabla] \cdot (\tilde{\mathrm{D}})^T \cdot [(\mathbf{B})^T - \nabla](\phi^*) - (A)^T(\phi^*) + (M)^T(\phi^*) + \sum_i (\beta_i F) C^{*i} = -\frac{\partial}{\partial t} \langle (V)^{-1}(\phi^*) \rangle$$

Henry gives arguments showing that the solutions of the above equations are the exact few-group direct and adjoint fluxes. He then presents the arguments that develop, from the above equations, the multigroup equations, the few-group equations with flux-adjoint-weighted constants, the few-group equations with flux-weighted constants, the overlapping energy-group equations, and the parallel-group equations.[10]

Kaplan, Davis, and Natelson[11] use angle-space synthesis as a method for transport approximations. Their method is of the P_N type, i.e., the true angular flux is approximated as a combination of trial functions whose combining coefficients are found by a variational principle. The deviation from standard procedures lies in the use of synthesis methods, with specifically tailored trial functions, to replace the standard orthogonal sets. This deviation is designed to obviate the large number of trial functions and large amount of computing time by changing the governing variational principle to canonical form and extending the admissible domain to include discontinuous functions.

The authors[11] divide the reactor into different spatial regions and approximate the true flux as a linear combination of trial functions chosen especially for the given region. The variational principle is then used to blend the discontinuous fluxes in the best manner. They begin their synthesis-development process by writing the monoenergetic transport theory in canonical form, using the odd and even equations:

$$\Omega \cdot \nabla \chi(\Omega,\mathbf{r}) + \Sigma(\mathbf{r})\, \dot{\psi}(\Omega,\mathbf{r}) = \int_{\Omega'} \Sigma_G(\Omega \cdot \Omega') \psi(\Omega',\mathbf{r}) d\Omega' + S_G(\Omega,\mathbf{r})$$

and

$$\Omega \cdot \nabla\psi(\Omega,\mathbf{r}) + \Sigma(\mathbf{r})\,\chi(\Omega,\mathbf{r}) = \int_{\Omega'} \Sigma_u(\Omega \cdot \Omega')\,\chi(\Omega',\mathbf{r})\,d\Omega' + S_u(\Omega,\mathbf{r})$$

where

$$S_G(\Omega) \equiv \frac{1}{2}\,[S(\Omega) + S(-\Omega)]$$

$$S_u(\Omega) \equiv \frac{1}{2}\,[S(\Omega) - S(-\Omega)]$$

$$\Sigma_G(\Omega \cdot \Omega') \equiv \frac{1}{2}\,[\Sigma_s(\Omega \cdot \Omega') + \Sigma_s(-\Omega \cdot \Omega')]$$

$$\Sigma_u(\Omega \cdot \Omega') \equiv \frac{1}{2}\,[\Sigma_u(\Omega \cdot \Omega') - \Sigma_u(-\Omega \cdot \Omega')]$$

$$\psi(\Omega) \equiv \frac{1}{2}\,[\phi(\Omega) + \phi(-\Omega)]$$

$$\chi(\Omega) \equiv \frac{1}{2}\,[\phi(\Omega) - (-\Omega)]$$

The introduction of linear operators allows the canonical transport equation to be written:

$$\Omega \cdot \nabla\chi + g_G\psi = S_G$$

$$\Omega \cdot \nabla\psi + g_u\chi = S_u$$

The authors[11] then apply a variational approach which is equivalent to that used for the transport problem, using the inner product notation for angular integration and noting that g_G and g_u are self-adjoint with respect to this inner product. Next they define the functional on the space of ordered pairs $\eta(\Omega,\mathbf{r})$, $\xi(\Omega,\mathbf{r})$ which are real-valued functions of direction and position in reactor region R:

$$F(\eta,\xi) = \int \left[\frac{1}{2}\langle\eta, g_G\eta\rangle - \frac{1}{2}\langle\xi, g_u\xi\rangle - \langle\eta, S_G\rangle + \langle\xi, S_u\rangle - \langle\xi, \Omega \cdot \nabla\eta\rangle\right] dR$$

$$+\frac{1}{2}\oint\left\{-\int_{\Omega\cdot\mathbf{n}<0} \eta[\eta - 2A(\Omega)](\Omega \cdot \mathbf{n})\,d\Omega + \frac{1}{2}\int_{\Omega\cdot\mathbf{n}>0} \eta[\eta - 2A(\Omega)](\Omega \cdot \mathbf{n})\,d\Omega\right\} dS$$

This functional has as its stationary point the pair of functions $\psi(\Omega,\mathbf{r}),\chi(\Omega,\mathbf{r})$. In order to extend the principles to admit discontinuous trial functions, the domain of the functional above must include discontinuous functions. The authors then specify values that the functional takes on at all points in the extended domain, arriving at a general expression for the functional.

To develop an approximation procedure, the authors partition the reactor region and approximate ψ and χ by:

$$\psi_{\text{approx.}}(\Omega,\mathbf{r}) = \sum_{k=1}^{K} G_k(\Omega)g_k(\mathbf{r})$$

$$\chi_{\text{approx.}}(\Omega,\mathbf{r}) = \sum_{m=1}^{M} U_m(\Omega)u_m(\mathbf{r})$$

where g_k and u_m are the mixing coefficients. An equivalent form of these equations is $\psi_{\text{approx.}} = G^T g$, $\chi_{\text{approx.}} = U^T u$, where the T superscript denotes transpose. Substituting these into the extended functional gives a reduced functional. Subsequent application of a variational principle yields the Euler equations of the reduced functional along with the associated boundary and continuity conditions. The Euler equations are then:

$$\langle G, \Omega U^T\rangle \cdot \nabla u + \langle G,\ g_G G^T\rangle g = \langle G, S_G\rangle$$

$$\langle u, \Omega G^T\rangle \cdot \nabla g + \langle U, g_u U^T\rangle u = \langle U, S_u\rangle$$

and the boundary conditions are:

$$\left[\int_{\Omega\cdot\mathbf{n}<0} (\Omega \cdot \mathbf{n})GG^T\,d\Omega\right]g + \left[\int_{\Omega\cdot\mathbf{n}>0} (\Omega \cdot \mathbf{n})GU^T\,d\Omega\right]u = \int_{\Omega\cdot\mathbf{n}<0} (\Omega \cdot \mathbf{n})GA\,d\Omega$$

The authors[11] use, as a numerical example, the monoenergetic Milne problem with isotropic scattering.

One might consider this approach to reactor problems as a combined modal analysis–variational approximation. The utility of the variational principles is that they will admit the discontinuous trial functions.

Buslik[12] has generalized the work of Davis by developing the variational principle to yield both upper and lower bounds for:

$$\langle S^*, \phi\rangle = \int S^*(\mathbf{r},\Omega)\,\phi(\mathbf{r},\Omega)\,d\mathbf{r}\,d\Omega$$

with S^* arbitrary rather than as Davis' $S^*(\mathbf{r},\Omega) = S(\mathbf{r},-\Omega)$. The essential difference in Buslik's treatment of the transport equation is a change of variable so that one-half of the sum and one-half of the difference of the direct and adjoint fluxes are used rather than the fluxes and adjoints themselves. That is,

instead of $\phi(\mathbf{r},\Omega)$ and $\phi(\mathbf{r},-\Omega)$ as variables, Buslik uses

$$\psi = \frac{\phi + \phi^*}{2}$$

and

$$\chi = \frac{\phi - \phi^*}{2}$$

Spherical-harmonics expansions are used to obtain the trial functions, and only conventional P_N calculations are required to obtain both even- and odd-parity components of the approximate function for the dependent variable, ψ. Rewriting the Boltzmann monoenergetic transport equation with the new ψ variable and applying boundary conditions yield a self-adjoint functional. Buslik establishes the positive definite property of the functional from the assumption of a macroscopic cross section that is greater than zero throughout a given region. From this, the validity of the new form of the transport equation is verified, so that

$$(H_s - H_a H_s^{-1} H_a)\psi = S_s - H_a H_s^{-1} S_a$$

where H is an operator given by $H = \Omega \cdot \nabla + G$ and G is the scattering and absorption operator.

Zweibel and Bowes[13,14] have applied space-angle synthesis to the criticality problem for a slab that is infinite in extent and finite in thickness and to the subcritical fixed-source problem. They choose the spatial trial functions for the Boltzmann transport equation and then determine the values for the angular functions and the spatial combining coefficients.

For the critical slab problem, vacuum boundary conditions are assumed and the functionals are chosen so that they are stationary around the eigenvalue to be determined. The chosen trial functions are inserted into the functional to give a reduced functional for which the Euler-Lagrange equations are developed. These equations are a pair of coupled integral equations that are readily solved because of the degenerate nature of the kernel. The solution to the integral equations gives the eigenvalue.

In the uniform-source problem the functionals are chosen as the neutron flux and adjoint, and the boundary conditions are applied. The trial functions are chosen and inserted into the Lagrangian, and then the spatial integrals are evaluated. The Euler-Lagrange equations are obtained from the reduced functional. The eigenvalue is obtained from the Euler-Lagrange equations, which are readily solved.

Buslik[15] presents a method for improving the spectra over which neutron cross sections are averaged in order to obtain the few-group constants for the diffusion equation. This method is a variational technique using spatial adjoint weighting to improve, by iteration, the trial spectra used in the overlapping-spectra method. He uses a variational principle that admits discontinuous trial functions, and he incorporates the interface flux-continuity conditions by Lagrange multipliers. He assumes few-group fluxes and adjoints and calculates the spectrum functions. The regions into which the reactor model has been divided are constructed so that the cross sections are all continuous with respect to the region. The Lagrange multipliers are defined for a given radius on the interface between any two regions and are selected so that they approximate the normal components of the direct and adjoint currents on the interface. Following the usual synthesis procedures, Buslik obtains a set of eigenvalue equations whose surface terms couple the spectra for the different regions.

NUMERICAL EXAMPLES

As the theory and methodology of variational principles and synthesis methods are adapted to reactor-physics calculations approximating the complex physical events that occur within the core, attention is slowly turning to detailed, specific examples that tend to illustrate the basic principles.

The synthesis of three-dimensional group-diffusion-theory fluxes from two-dimensional discontinuous base functions was reported by Yasinsky and Kaplan[16] in June 1967. The relative invariance of materials properties in the axial direction of the core lends itself to the use of these two-dimensional trial functions in the form:

$$\psi^g(x,y,z) = \sum_{k=1}^{K_g} H_k^g(x,y) z_k^g(z)$$

For large complex reactors with axially varying geometries, the authors make the trial functions discontinuous functions of z as well as x,y. They utilize axially discontinuous trial functions and start with the canonical form of the variational principle.

The continuity properties of both direct and adjoint functions are defined; the stationary points of the functional are quartets $(\psi,\psi^*,\mathbf{J},\mathbf{J}^*)$ which are defined so that ψ and ψ^* are the direct and adjoint

eigenfunctions and **J** and **J*** are the corresponding currents.[16]

Yasinsky and Kaplan then extend the domain of the functional to include quartets with only piecewise-continuous u,u^* such that the stationary points remain unchanged. The form for the extended functional then becomes:

$$F(u,u^*,\mathbf{v},\mathbf{v}^*) = \left(\int_R u^{*T} M u \, dR\right)^{-1} \times \left\{\sum_{j=1}^{J} \int_{R_j} (u^{*T} A u - \mathbf{v}^{*T} \cdot D^{-1} \mathbf{v} - \mathbf{v}^{*T} \cdot \nabla u - \mathbf{v}^{T} \cdot \nabla u^*) \, dR - \frac{1}{2} \oint_{S_{int}} [\mathbf{v}^*(+) + \mathbf{v}^*(-)][u(+) - u(-)] + [\mathbf{v}^T(+) + \mathbf{v}^T(-)][u^*(+) - u^*(-)] \right\} \cdot \mathbf{n} \, dS$$

Using a mesh structure on the z axis of the reactor which assigns material properties between mesh points so that $D(z) = D_n$, $A(z) = A_n$, and $M(z) = M_n$ for $z_n < z < z_{n+1}$, the authors define unit step functions so that

$$S_n(z) = \begin{cases} 1, & z_n - \dfrac{h_n}{2} < z < z_n + \dfrac{h_n}{2} \\ 0, & \text{otherwise} \end{cases} \qquad n = 1,2,\ldots,N-1$$

$$t_n(z) = \begin{cases} 1, & z_n < z < z_{n+1} \\ 0, & \text{otherwise} \end{cases} \qquad n = 0,1,2,\ldots,N-1$$

Now, let $F^g_{kn}(x,y)$, $k = 1, 2, \ldots, K_g$ denote a set of continuous functions of x,y which approximate the actual x,y shape of the flux ψ^g in the energy group g in the vicinity of axial elevation z_n. The term ψ^g is to be approximated as a linear combination of the terms F^g_{kn}, i.e.,

$$\psi^g(x,y,z) \approx \sum_{n=1}^{N-1} \sum_{k=1}^{K_g} s_n(z) F^g_{kn}(x,y) \phi^g_{kn}$$

where the terms ϕ^g_{kn} are the combining coefficients and K_g is the number of trial functions used which is in group g. The adjoint flux and direct and adjoint currents are chosen in the same manner as above.

Yasinsky and Kaplan[16] then arrange the trial functions $F^g_{kn}(x,y)$ into diagonal matrices and arrange the combining coefficients into columns; this procedure is then repeated for the adjoint flux and for the direct and adjoint currents. The following equations are the results:

$$\psi(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K} s_n(z) F_{kn}(x,y) \phi_{kn}$$

$$J_x(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K_x} s_n(z) [t_{n-1}(z) D_{n-1} + t_n(z) D_n] X_{kn}(x,y) \xi_{kn}$$

$$J_y(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K_y} s_n(z) [t_{n-1}(z) D_{n-1} + t_n(z) D_n] Y_{kn}(x,y) \eta_{kn}$$

$$J_z(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K_z} t_n(z) Z_{kn}(x,y) \zeta_{kn}$$

$$\psi^*(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K} s_n(z) F^*_{kn}(x,y) \phi^*_{kn}$$

$$J^*_x(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K_x} s_n(z) [t_{n-1}(z) D_{n-1} + t_n(z) D_n] X^*_{kn}(x,y) \xi^*_{kn}$$

$$J^*_y(x,y,z) \cong \sum_{n=1}^{N-1} \sum_{k=1}^{K_y} s_n(z) [t_{n-1}(z) D_{n-1} + t_n(z) D_n] Y^*_{kn}(x,y) \eta^*_{kn}$$

$$J^*_z(x,y,z) \cong \sum_{n=0}^{N-1} \sum_{k=1}^{K_z} t_n(z) Z^*_{kn}(x,y) \zeta^*_{kn}$$

Stacking both the diagonal matrices and combining coefficients for both the flux and current equations allow the equations to be written in a more compact notation. Substituting the reduced forms for the fluxes and currents into the functional produces a reduced functional with the appropriate boundary and stationary conditions. These equations are, of course, the synthesis equations.

To simplify the synthesis equations, the authors[16] select specific current trial functions. Three-point difference equations are obtained from the simplified synthesis equations; the neutron-flux combining coefficients, in turn, are obtained from the difference equations. Because of the similarities between the synthesis difference equations (the three-point difference equations) and the usual finite difference equations, the synthesis equations are readily solved by the usual methods of the diffusion codes.

To illustrate the use and applicability of the discontinuous synthesis techniques, Yasinsky and Kaplan give numerical examples, two of which are summarized below. The problems require synthesis of two-dimensional fluxes from one-dimensional trial functions with the trial functions being used as weight functions.

Yasinsky and Kaplan's problem 1 illustrates the ability to selectively switch trial functions along the axial dimensions. Figure 1 shows the geometry for the

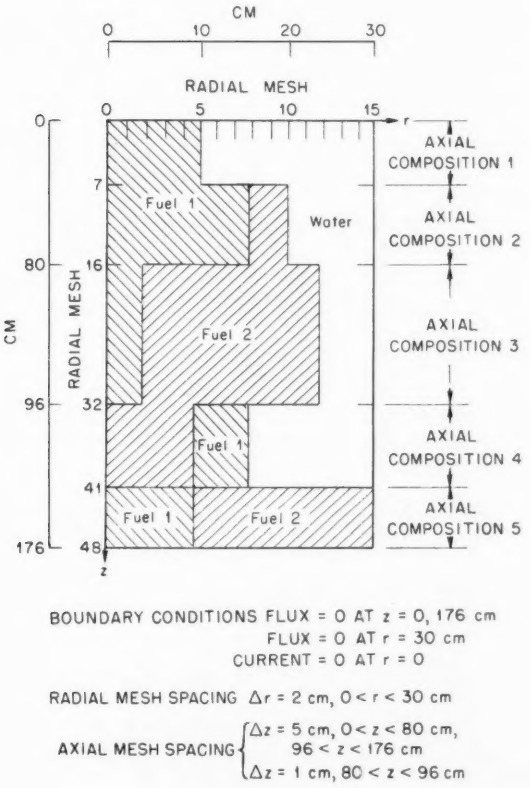


Fig. 1 Reactor geometry for test problem 1. (From Ref. 16.)

problem; this is a complicated cylindrical reactor whose two-group materials are given in Table 2. Figure 2 shows the results of the flux calculations.

Problem 2 was chosen to show the ability of the technique to synthesize problems in which the x,y boundaries of the reactor are functions of z. Figure 3 shows the reactor configuration; Table 3 lists the two-group materials constants. Figure 4 shows the results of a flux calculation.

Table 2 Two-Group Material Constants for Test Problem 1 (Ref. 16)

	Fuel 1	Fuel 2	Water
D^1, cm	1.69	1.77	1.65
Σ_a^1, cm^{-1}	0.0076	0.002	0.00037
Σ_r^1, cm^{-1}	0.015	0.018	0.038
$\nu\Sigma_f^1$, cm^{-1}	0.0068	0.003	0.0
D^2, cm	0.44	0.38	0.275
Σ_a^2, cm^{-1}	0.067	0.11	0.011
$\nu\Sigma_f^2$, cm^{-1}	0.071	0.12	0.0

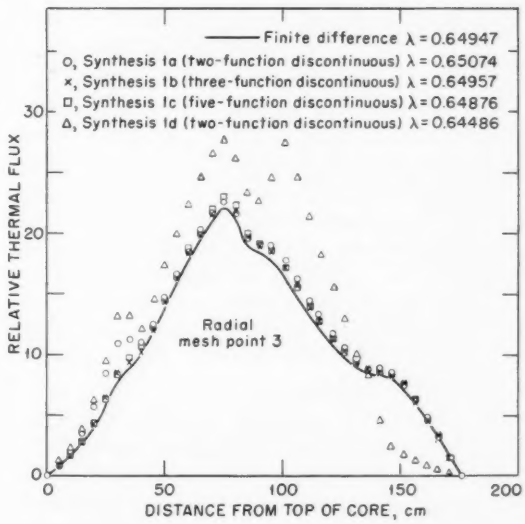


Fig. 2 Axial flux traverses for the thermal group—test problem 1. (From Ref. 16.)

Yasinsky[17] applies this same set of procedures for using discontinuous trial functions to the time domain. He approaches neutron-flux transient analyses using the space-time-dependent neutron-diffusion equations:

$$\nabla \cdot D\nabla\phi - A\phi - (1+\beta)\,\chi F^T\phi + \sum_{i=1}^{I} \lambda_i \chi_i C_i = V^{-1}\frac{\partial}{\partial t}\phi$$

and

$$\beta_i F^T\phi - \lambda_i C_i = \frac{\partial}{\partial t} C_i \qquad i = 1, 2, \ldots, I$$

where A = square absorption–removal matrix
F^T = transpose of fission–column matrix
D = diagonal diffusion–coefficient matrix
V^{-1} = diagonal inverse–group velocity matrix
χ = prompt–neutron–emission spectra

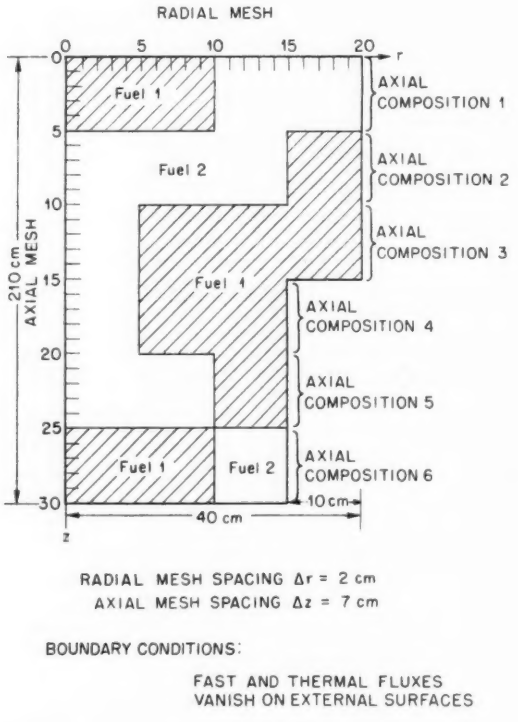


Fig. 3 Reactor core section for discontinuous-synthesis test problem 2. (From Ref. 16.)

Table 3 Two-Group Material Constants for Test Problem 2 (Ref. 16)

	Fuel 1	Fuel 2
D^1, cm	1.45	1.77
Σ_a^1, cm^{-1}	0.007	0.002
Σ_r^1, cm^{-1}	0.018	0.018
$\nu\Sigma_f^1$, cm^{-1}	0.003	0.003
D^2, cm	0.40	0.38
Σ_a^2, cm^{-1}	0.04	0.11
$\nu\Sigma_f^2$, cm^{-1}	0.04	0.12

The continuous time–synthesis method approximates the gth group flux as:

$$\phi g(\mathbf{r},t) = \sum_{k=1}^{K} \psi_k^g(\mathbf{r})\, T_k(t)$$

where ψ_k^g are known spatial trial functions. This discontinuous synthesis method optimizes, and thereby economizes, computing time by adding or deleting the spatial trial functions in the flux representations at specified times during a transient analysis.

Yasinsky's approach is to extend the continuous time–synthesis approximation to admit discontinuous functions. To do this, he builds a functional which admits time–discontinuous functions:

$$\begin{aligned}
F_1(\phi^*,\phi,C_i^*,C_i) = \sum_k \int_{\bar{T}_k} \iiint_R \Big\{ & -\nabla\phi^{*T}\cdot D\nabla\phi \\
& - \phi^{*T}[A-(1-\beta)\chi F^T]\phi \\
& - \frac{1}{2}\left(\phi^{*T}V^{-1}\frac{\partial}{\partial t}\phi - \phi^T V^{-1}\frac{\partial}{\partial t}\phi^*\right) \\
& \times \sum_{i=1}^{I}\Big[\phi^{*T}\lambda_i\chi_i C_i + C_i^*\beta_i F^T\phi - C_i^*\lambda_i C_i \\
& - \frac{1}{2}\left(C_i^*\frac{\partial}{\partial t}C_i - C_i\frac{\partial}{\partial t}C_i^*\right)\Big]\Big\}\, d\mathbf{r}\, dt \\
& - \frac{1}{2}\iiint_R \Big\{\phi^{*T}V^{-1}\phi - 2\phi^T V^{-1}\phi_f^*\Big)_{t=t_f} \\
& + (\phi^{*T}V^{-1}\phi - 2\phi^{*T}V^{-1}\phi_0)_{t=t_0} \\
& + \sum_{i=1}^{I}[(C_i^*C_i - 2C_iC_{if}^*)_{t=t_f} \\
& + (C_i^*C_i - 2C_i^*C_{i_0})]\Big\}\, d\mathbf{r} \\
& - \frac{1}{2}\sum_{t_k}\iiint_R\left(\frac{[\phi^*(+)+\phi^*(-)]^T}{2}V^{-1}\right. \\
& \times [\phi(+)-\phi(-)] - \frac{[\phi(+)-\phi(-)]^T}{2}V^{-1} \\
& \times [\phi^*(+)-\phi^*(-)] + \sum_{i=1}^{I}\left\{\frac{[C_i^*(+)+C_i^*(-)]}{2}\right. \\
& \times [C_i(+)-C_i(-)] \\
& \left.\left. - \frac{[C_i(+)+C_i(-)]}{2}[C_i^*(+)-C_i^*(-)]\right\}\right) d\mathbf{r}
\end{aligned}$$

The boundary conditions require that ϕ^*, ϕ, C_i^*, and $C_i \to 0$ on the external reactor surface S; $\phi_0(\mathbf{r})$ and $C_{i_0}(\mathbf{r})$ are the specified initial-flux matrix and ith-group precursor concentrations, respectively. The final values of the adjoint quantities for the flux and precursors are $\phi_j^*(\mathbf{r})$ and $C_i^*(\mathbf{r})$. Yasinsky then verifies the stationary conditions for the above discontinuous functional.

To illustrate the techniques of the time-discontinuous synthesis approximation, Yasinsky

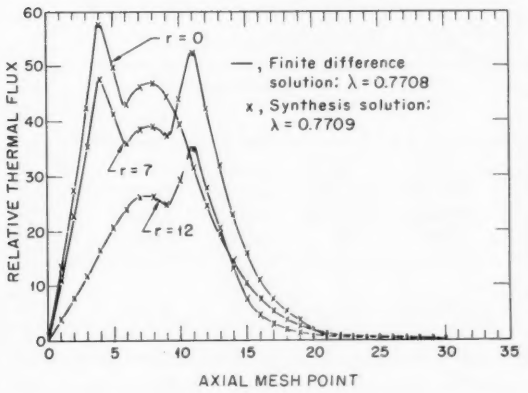


Fig. 4 Axial variation of thermal flux from discontinuous-synthesis test problem 2. (From Ref. 16.)

constructs a mesh structure in the time domain at time points $t_j (j = 0, 1, \ldots, N)$ and defines the intervals $\Delta^j = t_{j+1} - t_j$. He then introduces the following step functions:

$$\delta_j^*(t) = \begin{cases} 1, \text{ for } t_j \leqslant t < t_{j+1} \\ 0, \text{ otherwise} \end{cases}$$

$$\delta_j(t) = \begin{cases} 1, \text{ for } t - \theta\Delta^{j-1} \leqslant t < t + (1-\theta)\Delta^j \\ 0, \text{ otherwise} \end{cases}$$

$$\delta_{d_j}^{(t)} = \begin{cases} 1, \text{ for } t - \theta_d\Delta^{j-1} \leqslant t < t_j + (1-\theta_d)\Delta^j \\ 0, \text{ otherwise} \end{cases}$$

and determines the approximate forms for fluxes and precursors. These values for the fluxes and precursors are substituted into the functional, and the integrations are performed, yielding a reduced functional. The stationary conditions are formulated, and the expressions for the mixing coefficients, which are matrix difference equations, are derived.

Yasinsky[17] uses two test problems to show the ability of the time-discontinuous method for constructing approximate solutions to time-dependent difference equations from time-discontinuous sets of spatial trial functions. Both problems involve a 240-cm, initially critical slab reactor that is perturbed from critical in such a manner that the space-time effects are nonseparable.

One problem shows the effects of prompt-critical-type perturbation during 0.01 sec, with delayed-neutron effects being neglected. In the second problem the core is perturbed from critical by a ramp decrease in fast- and thermal-group absorption cross sections. The perturbation is such that the reactivity is always less than one-half prompt critical.

These problems illustrate the methods and results of the time-discontinuous synthesis techniques that permit the use of different sets of spatial trial functions in different time intervals in the solutions to the time-dependent group-diffusion equations.

Yasinsky[18] has also applied the time-synthesis technique to the analysis of space-time effects in large coupled-core reactors. The method proposed is a one-core trial-function procedure, i.e., one trial function per core for coupled cores; however, he is careful to recommend this for loosely coupled cores. The time-synthesis approximation procedure is the technique used, with the trial functions being easily obtained—even for loosely coupled cores. Yasinsky[18] uses three problems to illustrate the application: all are one-dimensional, two-energy groups with one effective group of delayed neutrons. One problem is representative of a fast coupled-core system, and the other two problems deal with a slab reactor.

Stacey[19] applies variational techniques to multichannel space-time synthesis of nonseparable reactor transients, using expansion functions that may be spatially discontinuous. The stationarity of this functional requires that:

1. The time-dependent P_1 equations and precursor balance, with adjoints, be satisfied.

2. The multigroup fluxes and currents, their adjoints, and the precursor densities be continuous with respect to time.

3. The multigroup flux, current, and adjoints be continuous with respect to space.

4. The boundary and initial conditions be satisfied.

To arrive at specific equations, Stacey defines axial- and planar-domain functions and time-domain functions. The notation in these definitions is then used to formulate expressions for the flux, current, and precursor concentrations:

$$\phi(x,y,z,t) = \sum_{n=1}^{N} \Delta_n \sum_{k=1}^{K} \Delta_k \sum_{i=1}^{I} \Delta_i \sum_{m=1}^{M} \psi^m_{nki}(x,y)\, \mathbf{a}^m_{nki}$$

$$\mathbf{J}(x,y,z,t) = \sum_{n=1}^{N} \Delta_n \sum_{k=1}^{K} \Delta_k \sum_{i=1}^{I} \Delta_i \left[\psi^m_{nki}(x,y)\mathbf{b}^m_{nki}(\mathbf{i}+\mathbf{j}) - \mathbf{D}_{nki}(x,y)\nabla\psi^m_{nki}(x,y)\,\mathbf{d}^m_i + \psi^m_{nki}(x,y)g^m_{nki}\mathbf{k}\right]$$

$$C^{\mu}(x,y,z,t) = \frac{\beta\mu}{\lambda\mu} \sum_{n=1}^{N} \Delta_n \sum_{k=1}^{K} \Delta_k \sum_{i=1}^{I} \Delta_{d_i} \sum_{m=1}^{M} \times F^T_{nki}(x,y)\psi^m_{nki}(x,y)\,\pi\,{}^{\mu}C^m_{nki} \qquad \mu = 1, 2, \ldots, 6$$

where ψ^m_{nki} is a $G \times G$ diagonal matrix whose elements are the $2D(XY)$ group fluxes for the expansion mode on in channel n and axial zone k during time interval i; $\mathbf{a}^m_{nki}$, b^m_{nki}, d^m_{nki}, and g^m_{nki} are $G \times 1$ column matrices whose elements are the combining coefficients for each energy group; ${}^{\mu}C^m_{nki}$ is a scalar expansion coefficient associated with ψ^m_{nki} in the expansion of precursor-density type m; π is a $G \times 1$ column matrix whose elements are unity; and ϕ is a $G \times 1$ column matrix of group fluxes and $\mathbf{J}$ is a $G \times 1$ column matrix of group currents. A similar expansion is made for the adjoint quantities.

Stacey then obtains the synthesis equations by substituting the direct and adjoint quantities for the flux and current into the original functional and applying the stationarity conditions. He compares numerical results obtained by single-channel and multichannel synthesis techniques for a slab reactor. Table 4 gives the prompt- and delayed-neutron characteristics, and Table 5 lists materials characteristics.

Table 4 Delayed- and Prompt-Neutron Parameters for Sample Problems[19]

Delayed-neutron fraction (β)	0.0075
Delayed-neutron decay constant (λ)	0.075 sec^{-1}
Inverse neutron velocity (v^{-1})	0.00001 $(cm/sec)^{-1}$

In Stacey's problem 1 the transient is initiated from critical by a ramp decrease in the macroscopic absorption cross sections, with no other nuclear properties being changed. The expansion functions are the static flux shapes; total power for one-, three-, four-, and five-channel synthesis calculations are compared with the exact RAUMZEIT calculation in Fig. 5. Figure 6 shows the flux distribution at 1.0 sec into the transient.

Table 5 Nuclear Properties of Materials in Sample Problems 1 and 2 (Ref. 19)

Problem 1 (Reflected Reactor)

	Material number			
	1	2	3	4
D	1.7309	1.5239	1.5251	1.5305
Σ_a	0.03882	0.01882	0.01871	0.02018
$\nu\Sigma_f$		0.01909	0.01877	0.02024
k_∞		1.0143	1.0032	1.0030

Problem 2 (Slab Reactor)

	Material number				
	1	2	3	4	5
D	1.4740	1.4740	1.4740	1.4740	1.4740
Σ_a	0.01866	0.01866	0.01866	0.01866	0.01866
$\nu\Sigma_f$	0.02200	0.02000	0.01900	0.01860	0.0220
k_∞	1.179	1.072	1.018	0.997	1.179

Stacey's problem 2 specifically illustrates the abilities of the variational multichannel synthesis techniques for treating nonseparable transients. The problem involves nonsymmetric perturbations for the model described in Table 5. Transient calculations were made with one-, three-, four-, and six-channel models. Figure 7 shows the ability of the multichannel methods to calculate tilted flux distributions and nonsymmetric shapes.

Stacey[19] summarizes the multichannel space-time variational model as one that assumes several flux shapes, defines average importance-weighted reaction probabilities and intermodal coupling based on these flux shapes, and develops a set of coupled equations that relate the expansion coefficients within different nodes. This technique contains features of both modal and nodal models and may be considered as an extension of either.

Yasinsky[20] has recently done further numerical studies on the applicability of combined space-time synthesis techniques to the three-dimensional time-dependent group-diffusion problem, using axially discontinuous trial functions. He expands the diffusion flux vector as:

$$\phi(x,y,z,t) \approx \sum_{k=1}^{K} H^z_k(x,y)\, Z_k(z,t)$$

where H^z_k are known x,y flux shapes witn a stepwise discontinuous z behavior. The trial functions are chosen to bracket the true solution along the z and t

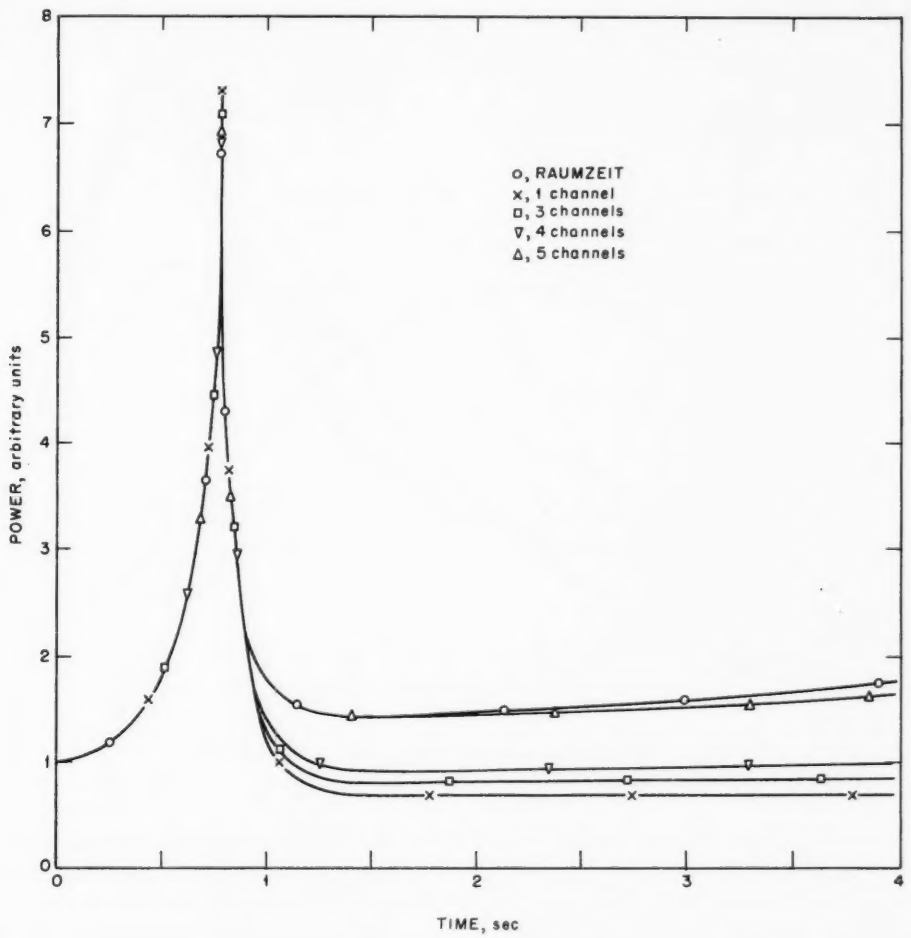


Fig. 5 Total power for transient. (From Ref. 19.)

dimensions. These trial functions are then blended by means of the unknown mixing coefficients $Z_k(z,t)$. The equations that define the axially discontinuous mixing coefficients $Z_k(z,t)$ are found by a variational principle.

Yasinsky's[20] numerical examples or problems are concerned basically with two-dimensional reactors that are described by two-group diffusion theory with one group of effective delayed neutrons. The trial functions $H_k(x)$ are spatially continuous one-dimensional diffusion solutions. The trial functions are also the weight functions.

The geometry Yasinsky uses for the first two examples is shown in Fig. 8; the materials are comparable to a large seed-blanket light-water breeder reactor (LWBR). The transients are caused by a perturbation of fast- and thermal-absorption and fast-removal cross sections in the crosshatched region of seed 1. In the calculation, the perturbed region extends from z mesh point 0 to 7; Σ_{a_1}, Σ_{a_2} and Σ_{r_1} are changed in a linear manner for 0.2 sec and then remain constant until $t = 1$ sec. The result is a delayed-critical problem (roughly equal to a control-rod insertion of 63 cm).

The initial flux distribution for the exact (TWIGL) solution was generated by PDQ-7, and the initial flux distribution for the synthesis solution was obtained using an experimental space-synthesis program. A time step of $t = 0.01$ sec was used for both the synthesis and exact solutions. Figures 9 to 11 compare the axial

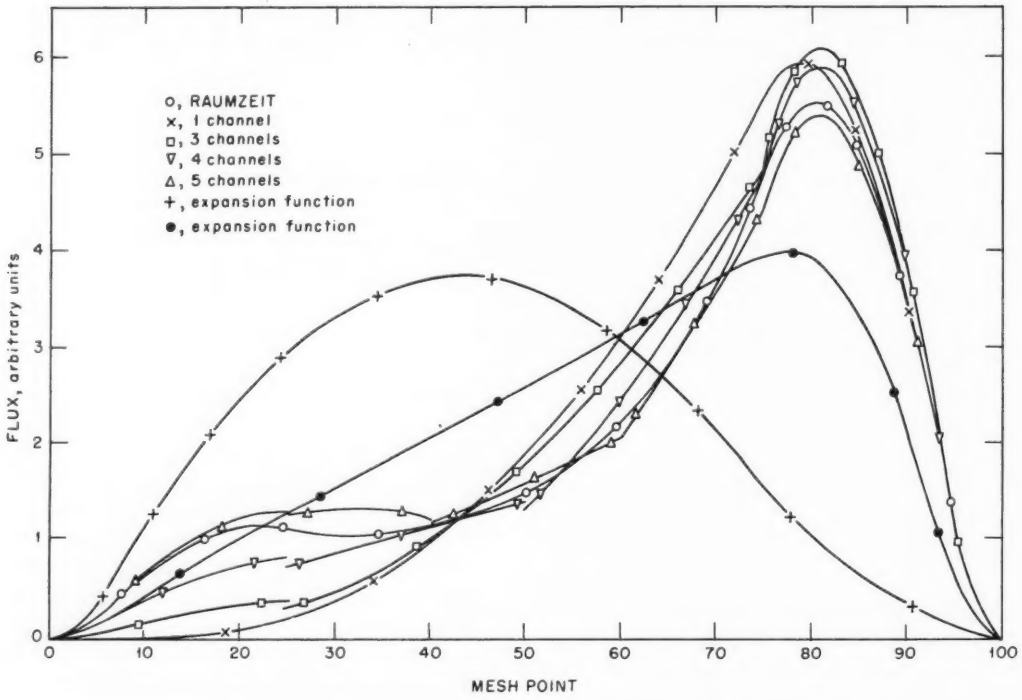


Fig. 6 Flux distribution at 1.0 sec for transient. (From Ref. 19.)

variation of the TWIGL and synthesis solutions at x mesh points 2, 8, and 12. Problem 2 is identical to problem 1 except that the seed 1 area is larger.

The next three problems are for a pressurized-water reactor (PWR) of cylindrical geometry (Fig. 12). The inverse group velocities are $1/v_1 = 1 \times 10^{-7}$(sec/cm) and $1/v_2 = 5 \times 10^{-6}$(sec/cm). One group of delayed neutrons with $\beta_i = 0.0064$ and $\lambda = 0.08$ is considered; zero-flux conditions are applied on all external surfaces. Again, as in problems 1 and 2, a set of three trial functions is used; these trial functions are the same one-dimensional static diffusion solutions. The first PWR problem (whose geometry is shown in Fig. 12) begins with the rods out of seed 3 region; the critical flux distribution for the exact solution is the PDQ-7 solution. The transient is introduced by means of a stepwise scram in the crosshatched seed 3 region. The result is an asymptotic shutdown reactivity of $\rho \sim 1.6\%$. The time step used in both the exact and synthesis solutions is 5×10^{-5} sec for $0 \leqslant t \leqslant 0.005$ sec and 0.01 sec thereafter. A comparison of the results of the exact and synthesis solutions is shown in Figs. 13 to 15.

These results show that the space-time synthesis technique is very accurate in describing a prompt-jump change in shape and level just after scram.

The next two PWR problems start in the critical state with the control rods in the crosshatched seed 3 region. The synthesis solution is constructed with the above trial functions. The initial flux for the exact solution of a rods-in configuration is taken from a critical PDQ-7 solution using the materials constants from Fig. 12 with the values for $\nu\Sigma_f$ being replaced by those in Table 6. The seed 3 region between z mesh points 12 and 14 is changed linearly from rods-in to rods-out condition (this simulates a 20-cm rod withdrawal resulting in a delayed-critical transient without significant shape changes). Figures 16 to 18 show the results.

These problems demonstrate the use of the combined space-time synthesis techniques using axially discontinuous trial functions. For transient synthesis solutions, Yasinsky reports a required 1.5 to 2 min of computing time per problem on a CDC-6600. The exact solutions required 4 to 10 min of computing time.

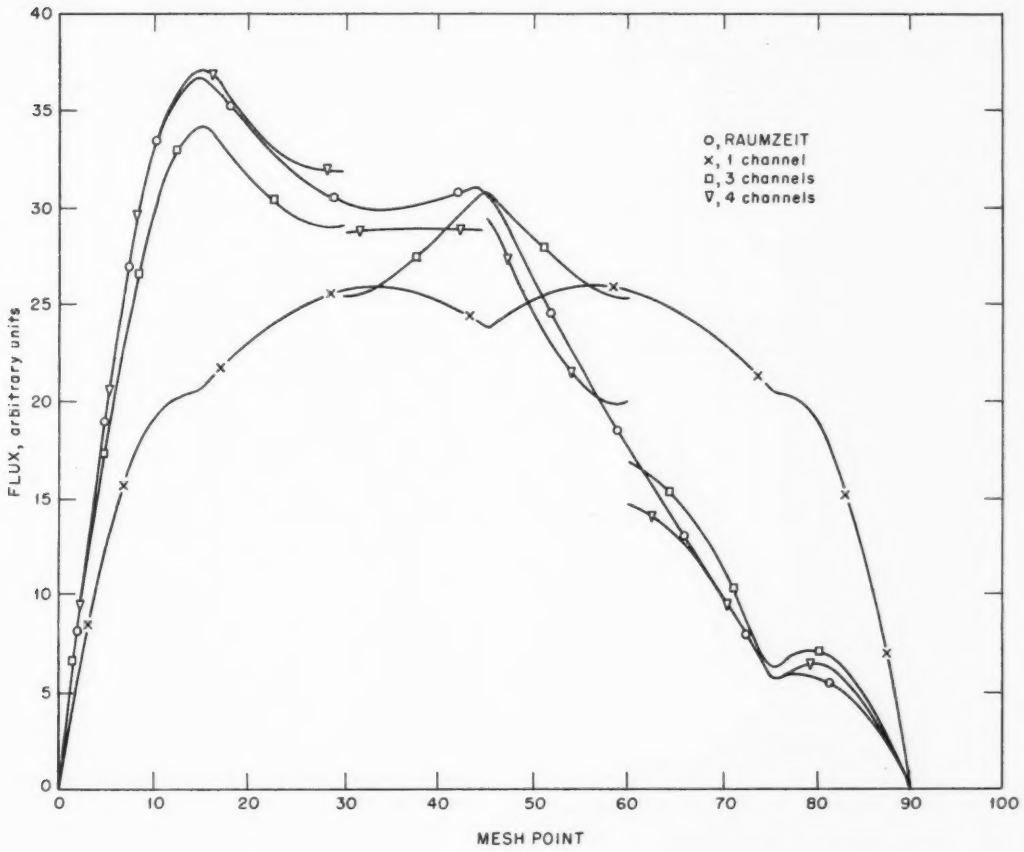


Fig. 7 Flux distribution at 0.5 sec for transient. (From Ref. 19.)

Becker[21] illustrates the use of discontinuous overlapping groups without the requirement of either staggered discontinuities or the additional complication of Lagrange multipliers. He writes the energy-dependent diffusion equation:

$$\nabla D\,\nabla\phi(\mathbf{r},E) - A\phi(\mathbf{r},E) = k^{-1}M\phi(\mathbf{r},E)$$

$$A\phi(\mathbf{r},E) = \Sigma_t\,\phi(\mathbf{r},E) - \int_0^\infty dE'\,\Sigma_s(E'\to E)\,\phi(\mathbf{r},E')$$

$$M\phi(\mathbf{r},E) = \chi(E)\int_0^\infty dE'\,\nu\Sigma_f(\mathbf{r},E')\,\phi(\mathbf{r},E')$$

and uses the flux-current formulation to obtain $D\mathbf{J}(\mathbf{r},e)+\nabla\phi(\mathbf{r},e)=0$ and the continuity equation $-\nabla\cdot\mathbf{J}(\mathbf{r},E) - A\phi(\mathbf{r},E) = k^{-1}M\phi(\mathbf{r},E)$. He then defines the operator $\Lambda = k^{-1}M - A$ which allows him to rewrite the above equations as:

$$\Lambda\phi(\mathbf{r},E) + \nabla\cdot\mathbf{J}(\mathbf{r},E) = 0$$

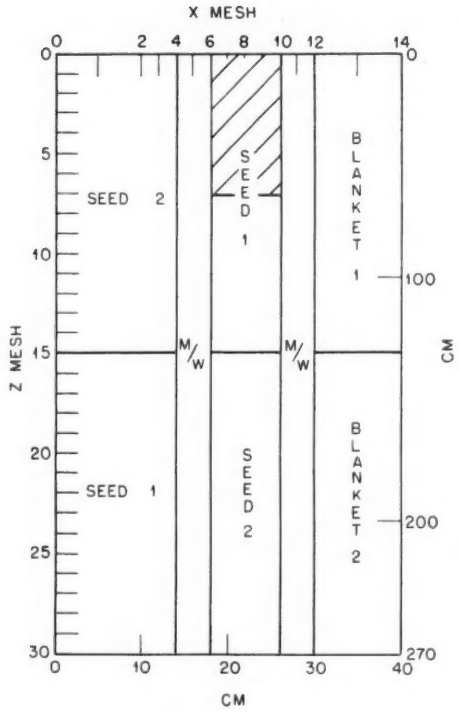
If only one-space dimension is used, the following functional can be constructed:

$$F = \int dE \int dx\left(\phi^*\Lambda\phi + \phi^*\frac{dJ}{dx} - J^*\frac{d\phi}{dx} - J^*D^{-1}J\right)$$

To illustrate the procedure, Becker uses a one-dimensional array of regions in slab geometry where the trial spectra are taken as the asymptotic spectra. For region B:

$$\phi_B(x,E) = \phi_{2B}(x)\,\psi_2(E) + \phi_{3B}(x)\,\psi_3(E)$$

i.e., at any interface, the spectrum is assumed to be an interpolation between asymptotic spectra. He examines both the discontinuity arising at an interface and the



Cross-Hatched Region

$\Delta\Sigma_{a_1} = -0.002$	$\beta = 0.0064$
$\Delta\Sigma_{r_1} = +0.001$	$\lambda = 0.08$
$\Delta\Sigma_{a_2} = -0.08$	$1/v_1 = 1 \times 10^{-6}$
	$1/v_2 = 5 \times 10^{-5}$

	Seed 1	Seed 2	Blanket 1	Blanket 2	Metal: Water
D_1	1.39	1.37	1.19	1.19	1.41
Σ_{a_1}	0.019	0.014	0.01	0.01	0.00093
Σ_{r_1}	0.006	0.009	0.007	0.007	0.019
$\nu\Sigma_{f_1}$	0.019991	0.013327	0.004665	0.003998	0
D_2	0.52	0.58	0.70	0.70	0.67
Σ_{a_2}	0.43	0.23	0.15	0.13	0.013
$\nu\Sigma_{f_2}$	0.573074	0.266546	0.099955	0.066637	0

Fig. 8 Geometry for LWBR-type problems. (From Ref. 20.)

interface conditions imposed by the functional. The description of the interface discontinuity is achieved through a delta function, to which is applied a symmetry condition. This symmetrization allows formulation of the direct and adjoint flux and current equations.

When the usual synthesis procedures of trial-function substitution into the functional are followed,

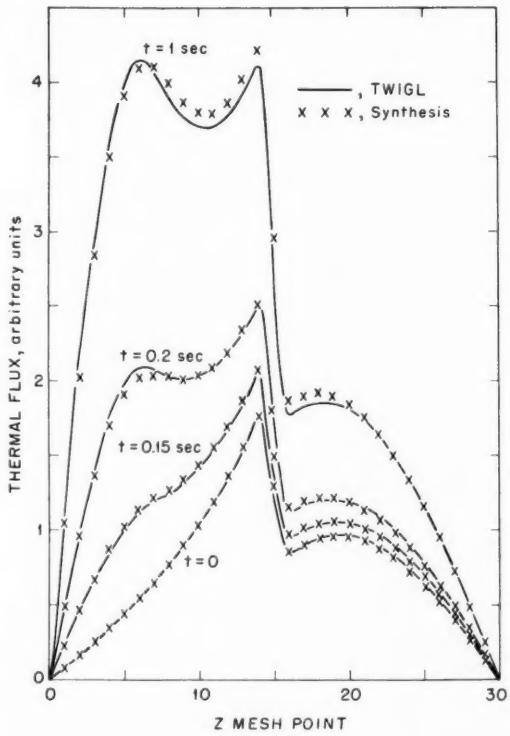


Fig. 9 Thermal flux at x mesh point 2, LWBR problem 1. (From Ref. 20.)

the results of an overdetermination of continuity conditions are revealed. Becker then addresses himself to the methods for avoiding these inconsistencies. He assumes that the region under consideration is sufficiently large so that the asymptotic spectrum tends to a steady value in the interior of the region. This assumption allows any variation at an interface to be considered small, and thus Becker arrives at the stationarity conditions for the functional. He briefly discusses the problems of multidimensionality also.

Pomraning[22] applies indirect variational methods to the determination of neutron-flux ratios in critical systems. He starts with the eigenvalve problem:

$$A(x,y)\psi_j(x,y) = \Lambda_j B(x,y)\psi_j(x,y)$$

where A and B are linear operators, ψ_j is the jth eigenfunction, and Λ_j is the corresponding eigenfunction. He then constructs the functional

$$F(\phi_j,\theta_j,\lambda_j) = G(\theta_j) + \int dx \int dy\, \theta_j(A\theta_j - \lambda_j B\phi_j)$$

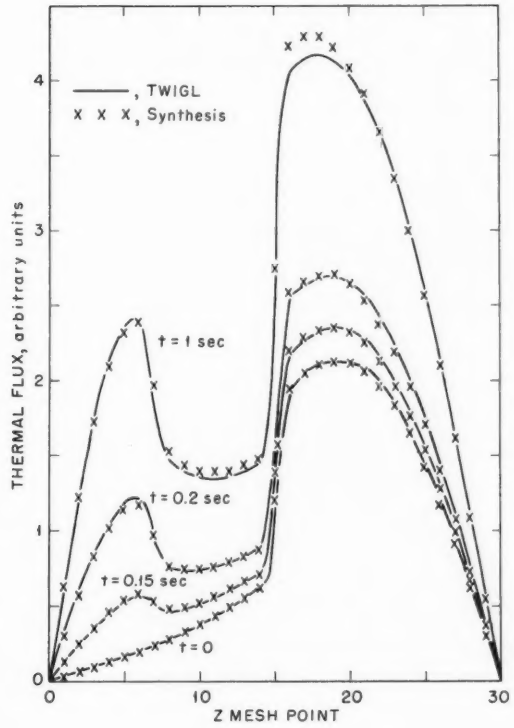


Fig. 10 Thermal flux at x mesh point 8, LWBR problem 1. (From Ref. 20.)

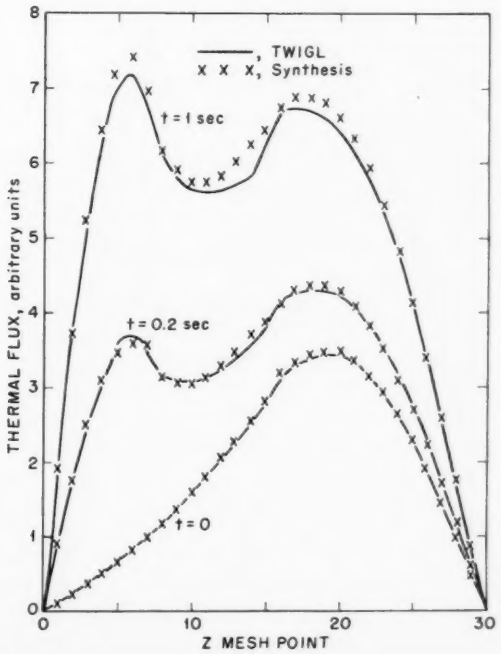
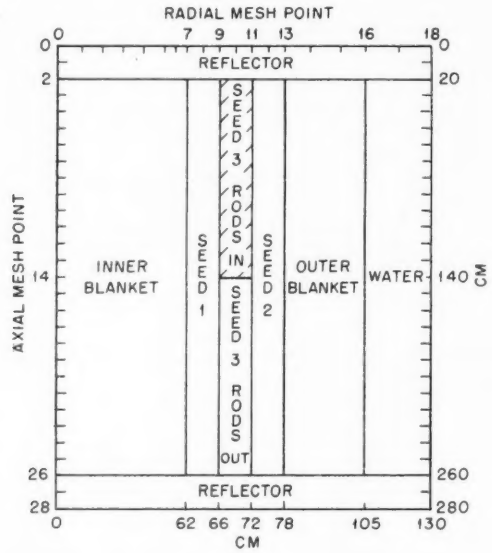


Fig. 11 Thermal flux at x mesh point 12, LWBR problem 1. (From Ref. 20.)



	Seed 1	Seed 2	Seed 3 (rods in)	Seed 3 (rods out)	Inner Blanket	Outer Blanket	Reflector	Water
D_1	1.367	1.404	1.377	1.377	1.291	1.297	1.412	1.625
Σ_{a_1}	0.01099	0.00661	0.01061	0.006551	0.008494	0.008400	0.001483	0.000365
Σ_{r_1}	0.01309	0.01582	0.01309	0.01586	0.01116	0.01114	0.005923	0.03797
$\nu\Sigma_{f_1}$	0.007142	0.006602	0.006046	0.006046	0.002677	0.002681	0	0
D_2	0.3825	0.3778	0.3384	0.3384	0.4565	0.4572	0.7162	0.2750
Σ_{a_2}	0.1511	0.09493	0.1339	0.09039	0.04975	0.04727	0.006549	0.01083
$\nu\Sigma_{f_2}$	0.194451	0.147798	0.171219	0.171219	0.058203	0.054510	0	0

Fig. 12 PWR cylindrical geometry for synthesis problems. (From Ref. 20.)

so that it gives a variational estimate of $G(\psi_j)$, which is an estimate of second order if θ_j and λ_j are first-order approximations to ψ_j and λ_j, respectively, and if θ_j is a first-order estimate of a certain solution of the derived auxiliary equations.

Pomraning then applies indirect variational methods to arrive at the trial functions needed to compute a variational estimate of a homogeneous functional for the eigenvalue equation. To illustrate the procedure, he uses a problem having one dependent and two independent variables with separable trial functions.

He shows that, when the indirect variational method is used to generate trial functions for both the eigenfunction and adjoint, as well as the trial eigenvalue, no difficulties arise. The resulting Euler-Lagrange equations of the reduced functional, including the constraint, have a unique, well-defined solution.

He also shows[22] that, if approximate eigenfunctions and eigenvalues are known from other sources and the indirect variational method is used to generate only an approximate adjoint, the method fails completely.

Lancefield[23] uses the overlapping-group method to obtain a more accurate description of fluxes at large distances from the core and to obtain reasonable flux approximations across interfaces such as core-reflector interfaces. The method is applied to a simple two-region reactor consisting of core and reflector. Results were compared with 16-group S_n calculations using the ANISN code. He reports reactivity estimates with 0.1% accuracy and agreement between estimated fluxes and reaction rates to within a few percent except for high-energy neutrons in the reflector. He had to use an alternative variational principle to improve agreement in the reflector; discontinuous trial functions were necessary to adequately represent the flux throughout the reactor over the whole energy range.

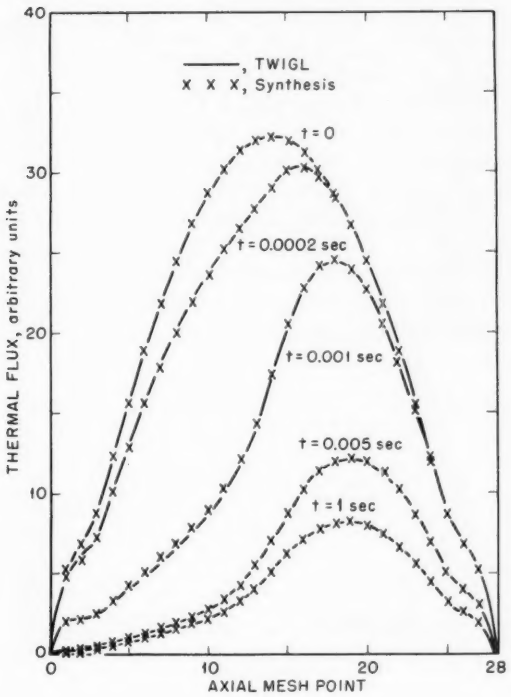


Fig. 13 Thermal flux at radial mesh point 5, PWR problem 1. (From Ref. 20.)

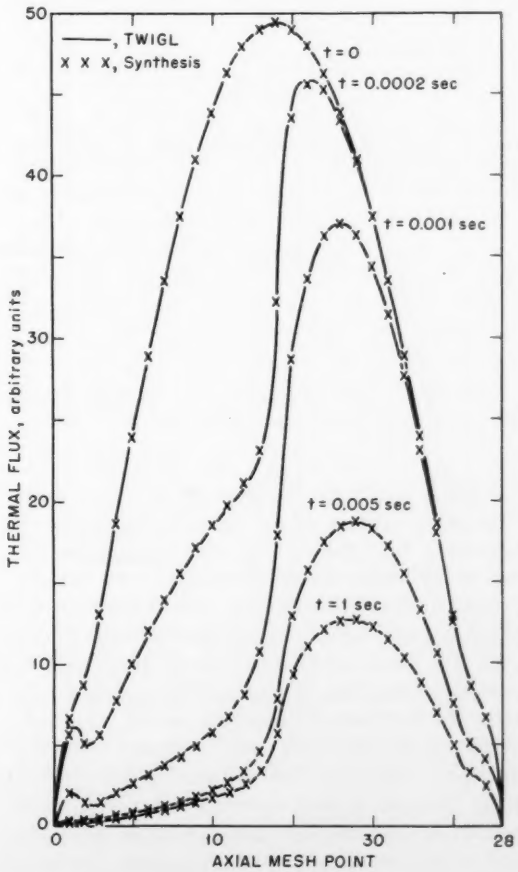


Fig. 14 Thermal flux at radial mesh point 10, PWR problem 1. (From Ref. 20.)

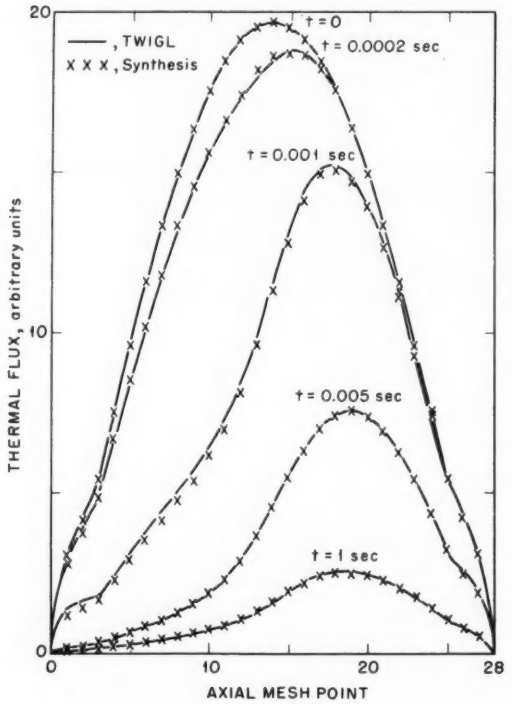


Fig. 15 Thermal flux at radial mesh point 15, PWR problem 1. (From Ref. 20.)

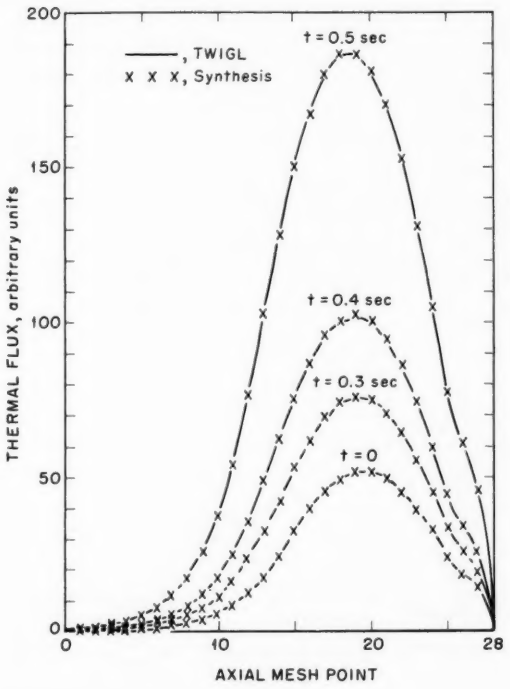


Fig. 16 Thermal flux at radial mesh point 5, PWR problem 2. (From Ref. 20.)

Table 6 Critical Fission Cross Sections for Rods-In PWR Configuration[20]

	Seed 1	Seed 2	Seed 3 (rods in)	Seed 3 (rods out)	Inner blanket	Outer blanket
$\nu\Sigma_{f_1}$	0.007258	0.006710	0.006145	0.006145	0.002721	0.002725
$\nu\Sigma_{f_2}$	0.197620	0.150206	0.174009	0.174009	0.059152	0.055399

Adams and Stacey[24] report an intrinsic anomaly observed in a flux-weighted collapsed-group synthesis calculation for a three-energy-group cylindrical reactor model having a single annular control rod. The expansion functions used were one-dimensional flux calculations of rodded and unrodded elevations in the core. The usual synthesis procedures were used with straightforward flux weighting. The numerical solutions for the synthesis equations, which are z and t dependent in final form, were compared with finite difference solutions. The collapsed-group synthesis static solutions yielded eigenvalues that were several percent in error and flux distributions in the rodded area of the core that were partially negative. The use of the straightforward flux weighting in the collapsed-group synthesis equations caused an overemphasis on the fast-group balance equation, and, consequently, the other two-group balance equations were underemphasized. Reduction of the fast-group scaling factor resulted in correct solutions to the synthesis equations. The authors elaborate on these points.

Neuhold and Ott[25] have used reaction rates rather than trial functions or adjoints to derive space-energy synthesis equations for fast reactors. These equations were obtained for a 1000-Mw(e) sodium-cooled fast breeder reference-reactor design. Results obtained from the use of reaction rates are compared with results from equations obtained using trial functions.

CONCLUSION

Variational methods for the treatment of space–, space–time–, or space–energy–dependent diffusion or transport equations that describe reactor core neutronics have been summarized to provide a composite history of the 1966–1969 activities in this area of reactor physics. In addition, it is my intent that these summaries be complete enough to allow the reader to draw his own conclusions as to areas where further developments or expositions might be useful, especially with regard to fast reactor physics and the physics of the large [on the order of 1000–Mw(e)] complex reactors.

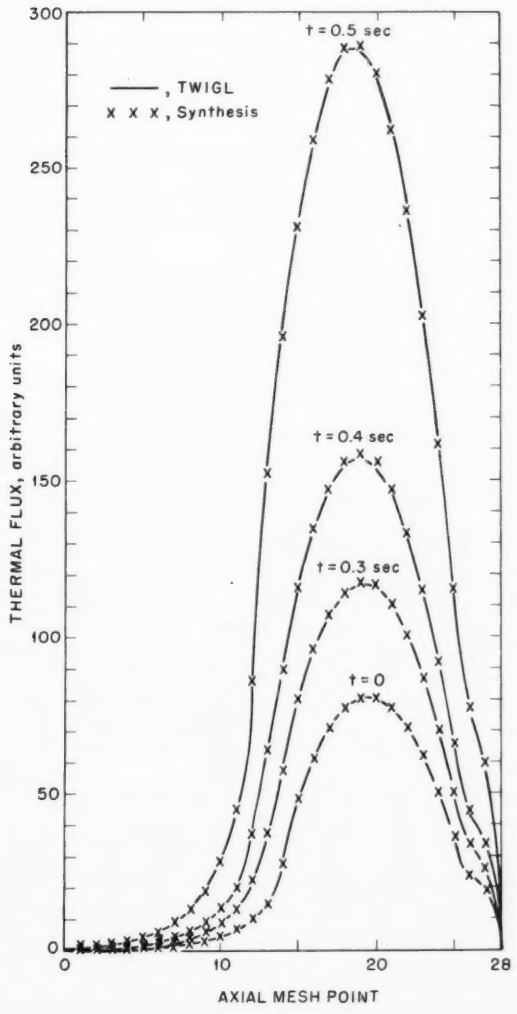


Fig. 17 Thermal flux at radial mesh point 10, PWR problem 2. (From Ref. 20.)

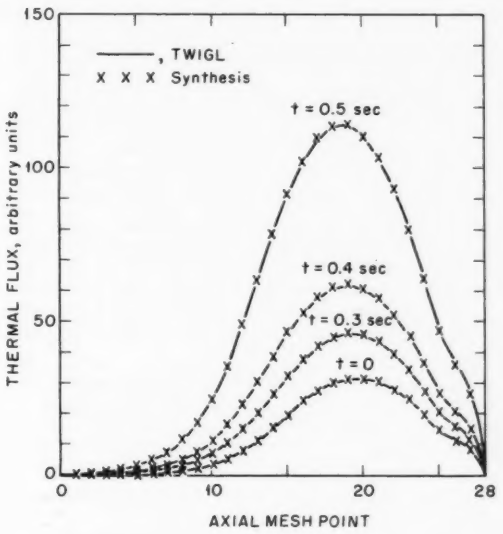


Fig. 18 Thermal flux at radial mesh point 15, PWR problem 2. (From Ref. 20.)

The sample problems included here were selected to illustrate the type of work that has been done and the results of this work. An overall organizational format for each of the summaries can be outlined as: presentation of the original equations used by each writer, the functional he assumed, and the resulting synthesis equations. Of course, there are some exceptions to this format; I hope this will not cause difficulties or confusion. The use of equations has been minimized; where possible, verbal summaries or descriptions have been included to make the overall text more readable.

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Suggested Reading List

Given below are references to material that appeared before 1966 on the variational principle or on synthesis techniques, or on both. This list is provided in case the reader would like to further acquaint himself with the topic for which I have attempted to summarize the literature over the last 3 years.

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4 **Noise Analysis in Nuclear Systems (TID-7679),**
University of Florida, 1964

5 **Radioactive Fallout from Nuclear Weapons Tests (CONF-765),**
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7 **Neutron Dynamics and Control (CONF-650413),**
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Presence and Persistence of Radionuclides; Possible Countermeasures Against Harmful Consequences of Radionuclide Contamination
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UNIVERSITIES, NATIONAL LABORATORIES, AND MAN'S ENVIRONMENT

Argonne Universities Association Conference
July 27–29, 1969, Chicago, Illinois

The growing nationwide concern about environmental pollution prompted Argonne Universities Association, a group of 30 universities which is responsible for determining the policies of Argonne National Laboratory, to sponsor this conference. Some 300 participants from universities, government, and industry met to consider environmental problems and their solution under the conference's two subtitles: Is Man on His Way to Extinction—Is It a Natural Process and Organizing to Understand and Shape Man's Environmental Interactions.

In announcing the conference, Dr. Philip N. Powers of Purdue University, President of Argonne Universities Association, said: "There is an urgent need for all 'knowledge' institutions to seek the solution of environmental problems. . . . It is important that universities and national laboratories refashion themselves to attack these problems. We hope to learn how this can be done by exchanging ideas and by learning from the experiences of experts in academic planning and pollution control."

The major topic of discussion at the meeting was the desirability and feasibility of establishing a socio-technological research organization, utilizing the physical and intellectual resources of the universities and of Argonne National Laboratory, to study large-scale environmental problems.

The Atomic Energy Commission, keenly aware of the difficult decisions involved in exploiting technology to better human life while at the same time protecting the natural environment, published these proceedings as a manifestation of its concern that the public be well informed and decide wisely. (167 pages, 23 figures)

PRINCIPAL SPEAKERS

RENE DUBOS, Department of Environmental Biomedicine, Rockefeller University, and Pulitzer Prize winning author

ELVIS J. STAHR, President, National Audubon Society

MELVIN PRICE, Representative from Illinois; Chairman, Subcommittee on Research, Development, and Radiation, Joint Committee on Atomic Energy

JOHN L. BUCKLEY, Office of Science and Technology, Executive Office of the President

JAMES T. RAMEY, Commissioner, U. S. Atomic Energy Commission

LAURA FERMI, Author and cofounder of one of the first citizens' committees to fight air pollution

CHET HOLIFIELD, Representative from California; Chairman, Joint Committee on Atomic Energy

HENRY S. ROWEN, President, The Rand Corporation

FRED H. HARRINGTON, President, University of Wisconsin

LAWRENCE R. HAFSTAD, Member of the National Academy of Engineering

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